Ground-state properties of even and odd Magnesium isotopes in a symmetry-conserving approach

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Even-Even Nuclei

A important development: describe even-even nuclei with effective interactions (Skyme, Gogny and relativistic ones). It mainly achieved by the means of **beyond-mean-field-theories** (BMFT). The shape parameters (β, γ) were used as coordinates in **GCM** and particle-number(**PN**) and angular-momentum(**AM**) symmetries were recovered by projection.

Odd-Even Nuclei

Odd nuclei are far more complicated to deal with (even at mean-field level like **HFB** or **BCS**). Furthermore, the blocked structure of the wave function entail the breaking of the **time-reversal** symmetry and **triaxial calculations** must be performed.

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Purpose

Report on the first systematic description of the odd and even nuclei of an isotopic chain in a symmetry-conserving approach with the **Gogny force** in a BMFT considering the (β, γ) degrees of freedom explicitly and dealing optimally with the pairing correlations.

The HFB wavefunction $|\phi\rangle$ is a product of quasi-particles α_ρ:

$$\alpha_\rho^\dagger = \sum_\mu U_{\mu\rho} c_\mu^\dagger + V_{\mu\rho} c_\mu \tag{1}$$

In their approach they have imposed three discrete self-consistent symmetries on basis states $c_\mu^\dagger, c_\mu$: spatial parity, $\hat{P}$, simplex, $\Pi_1 = \hat{P} e^{-i\pi J_x}$ and $\Pi_2 \Gamma$ with $\Pi_2 = \hat{P} e^{-i\pi J_y}$ and Γ the time reversal operator. The basis is symmetrized in such a way:

$$\Pi_1 c_k^\dagger \Pi_1^\dagger = +i c_k^\dagger, \quad \Pi_1 c_{\bar{k}}^\dagger \Pi_1^\dagger = -i c_{\bar{k}}^\dagger \tag{2}$$

with $k = 1, \ldots, M$ and $2M$ the dimension of the configuration space.

Using Latin indices to distinguish the levels according to their simplex, $\{k, l, m\}$ for simplex $+i$ and $\{\bar{k}, \bar{l}, \bar{m}\}$ for simplex $-i$. If the intrinsic wave function is an eigenstate of the simplex operator, for a paired even-even nucleus half of the quasiparticle operators $\alpha_\mu^\dagger$, have simplex $+i$ and the other half have simplex $-i$.

$$
\begin{aligned}
\alpha_m^\dagger &= \sum_{k=1}^{M} U_{km}^{+} c_k^\dagger + V_{km}^{+} c_{\bar{k}} \\
\alpha_{\bar{m}}^\dagger &= \sum_{k=1}^{M} U_{km}^{-} c_{\bar{k}}^\dagger + V_{km}^{-} c_k
\end{aligned}
\tag{3}
$$

with $m = 1, \ldots, M$

The wave function of ground state of an even-even nuclei:

$$|\phi\rangle = \prod_{\mu=1}^{2M} \alpha_\mu |-\rangle \tag{4}$$

The quasiparticle vacuum:

$$\alpha_\mu |\phi\rangle = 0, \mu = 1, \ldots, 2M. \tag{5}$$

The one quasiparticle excitations (correspond to odd-even nuclei):

$$|\tilde{\phi}\rangle = \alpha^\dagger_{\rho_1} |\phi\rangle \tag{6}$$

They can be written as vacuum to the quasiparticle operators $\tilde{\alpha}_\rho$,

$$\tilde{\alpha}_\rho |\tilde{\phi}\rangle = 0, \rho = 1, \ldots, 2M. \tag{7}$$

$\{\tilde{\alpha}^{\dagger}_{\rho}\}$ are obtained from the set $\{\alpha^{\dagger}_{\mu}\}$ by replacing $\alpha^{\dagger}_{\rho_1}$ by α_{ρ_1}. The simplex state $|\tilde{\phi}\rangle$ is given by $\Pi_1|\tilde{\phi}\rangle = i^n|\tilde{\phi}\rangle$ ($n=1$ if $\alpha^{\dagger}_{\rho_1}$ has simplex $+i$ and $n=-1$ if $\alpha^{\dagger}_{\rho_1}$ has simplex $-i$). The blocked wave function $|\tilde{\phi}\rangle$ is vacuum to $M_+ = M - n$ operators $al\tilde{p}ha^{\dagger}_m$ with simplex $+i$ and to $M_- = M + n$ operators $\tilde{\alpha}^{\dagger}_{\bar{m}}$ with simplex $-i$.

$$\begin{aligned} \tilde{\alpha}^{\dagger}_m &= \sum_{k=1}^{M} \tilde{U}^{+}_{km} c^{\dagger}_k + \tilde{V}^{+}_{km} c_{\bar{k}}, m = 1, \ldots, M_+, \\ \tilde{\alpha}^{\dagger}_{\bar{m}} &= \sum_{k=1}^{M} \tilde{U}^{-}_{km} c^{\dagger}_{\bar{k}} + \tilde{V}^{-}_{km} c_k, m = 1, \ldots, M_-. \end{aligned} \tag{8}$$

The matrices $(\tilde{U}^+, \tilde{V}^+, \tilde{U}^-, \tilde{V}^-)$ are obtained from (U^+, V^+, U^-, V^-) by corresponding columns exchange

Constrain calculation:

$$E'[\tilde{\phi}] = \frac{\langle\tilde{\phi}|\hat{H}\hat{P}^N|\tilde{\phi}\rangle}{\langle\tilde{\phi}|\hat{P}^N|\tilde{\phi}\rangle} - \langle\tilde{\phi}|\lambda_{q_0}\hat{Q}_{20} + \lambda_{q_2}\hat{Q}_{22}|\tilde{\phi}\rangle \tag{9}$$

Lagrange multipilier γ_{q_0} and γ_{q_2} being determined by the constraints

$$\langle\tilde{\phi}|\hat{Q}_{20}|\tilde{\phi}\rangle = q_0, \langle\tilde{\phi}|\hat{Q}_{22}|\tilde{\phi}\rangle = q_2 \tag{10}$$

GCM:

$$\begin{aligned} |\Psi_{M,\sigma}^{N,I,\pi}(\beta,\gamma)\rangle &= \sum_K g_{K\sigma}^I P^N P_{MK}^I |\tilde{\phi}^\pi(\beta,\gamma)\rangle \\ &= \sum_K g_{K\sigma}^I |IMK,\pi,N,(\beta,\gamma)\rangle \end{aligned} \tag{11}$$

Reduced Hill-Wheeler-Griffin equation

$$\sum_{K'} (\mathcal{H}_{K,K'}^{N,I,\pi} - E_\sigma^{N,I,\pi} \mathcal{N}_{K,K'}^{N,I,\pi}) g_{K'\sigma}^I = 0 \tag{12}$$

Hamiltonian and norm overlaps:

$$\begin{aligned} \mathcal{H}_{K,K'}^{N,I,\pi} &= \langle IMK,\pi,N,(\beta,\gamma)|H|IMK',\pi,N,(\beta,\gamma)\rangle \\ \mathcal{N}_{K,K'}^{N,I,\pi} &= \langle IMK,\pi,N,(\beta,\gamma)|IMK',\pi,N,(\beta,\gamma)\rangle \end{aligned} \tag{13}$$

Energy

$$E_{\sigma}^{N,I,\pi}(\beta,\gamma) = \frac{\langle \Psi_{M,\sigma}^{N,I,\pi}(\beta,\gamma)|H|\Psi_{M,\sigma}^{N,I,\pi}(\beta,\gamma)\rangle}{\langle \Psi_{M,\sigma}^{N,I,\pi}(\beta,\gamma)|\Psi_{M,\sigma}^{N,I,\pi}(\beta,\gamma)\rangle} \quad (14)$$

The collective wave function

$$G_{K,\sigma}^{J} = \sum_{K'} (\mathcal{N}^{N,I,\pi})_{K,K'}^{1/2} g_{K',\sigma}^{I} \quad (15)$$

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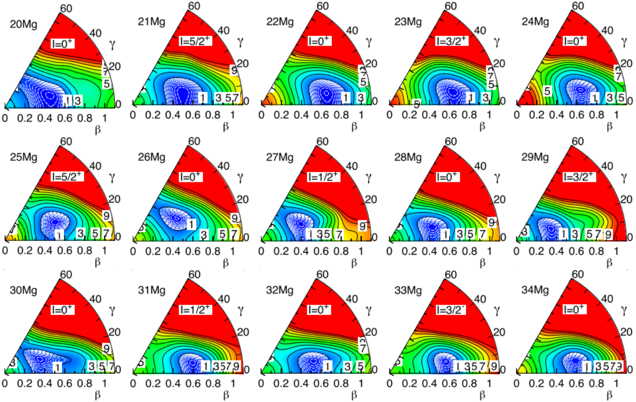

Fig. 1. Contour plots of $E^{N,I,\pi}_{\sigma=0}(\beta,\gamma)$, see Eq. (13), as a function of (β,γ) for positive parity and for the angular momentum I providing the lowest energy. The solid black contour lines start at 1 MeV and increase 1 MeV. The dashed white lines start at zero and increase 0.1 MeV. The zero contour is only present if the minimum is flat enough. The angle γ units are degrees.

Table 1
The 2nd and 3rd columns display the spin and parity and the β, γ deformations of the ground state of the different isotopes. Notice that only ^{33}Mg has a ground state with negative parity. The 4th column shows the experimental β deformation taken from Refs. [42,43]. The 5th column lists the $|K|$ component with the largest weight in the wave function, see Eq. (17), with the percentage of this $|K|$ value in the total wave function. The 6th column provides the theoretical spectroscopic quadrupole moments, in efm^2.

A	I^π	β, γ	β_{exp}	$\|K\|(\%)$	Q_{spec}
20	0^+	0.46, 17.5°	–	–	–
21	$\frac{5}{2}^+$	0.54, 14.9°	–	$\frac{1}{2}$(99.1%)	−17.80
22	0^+	0.65, 12.2°	0.58 (11)	–	–
23	$\frac{3}{2}^+$	0.64, 10.9°	–	$\frac{3}{2}$(99.9%)	13.89
24	0^+	0.65, 12.2°	0.605 (8)	–	–
25	$\frac{5}{2}^+$	0.54, 17.5°	–	$\frac{5}{2}$(99.7%)	22.47
26	0^+	0.49, 25.3°	0.482 (10)	–	–
27	$\frac{1}{2}^+$	0.41, 23.4°	–	$\frac{1}{2}$(100%)	0
28	0^+	0.46, 17.5°	0.491 (35)	–	–
29	$\frac{3}{2}^+$	0.37, 19.1°	–	$\frac{1}{2}$(96.0%)	−10.71
30	0^+	0.39, 21.1°	0.431 (19)	–	–
31	$\frac{1}{2}^+$	0.60, 11.7°	–	$\frac{1}{2}$(100.0%)	0
32	0^+	0.54, 14.9°	0.473(43)	–	–
33	$\frac{3}{2}^-$	0.60, 11.7°	–	$\frac{3}{2}$(99.9%)	14.17
34	0^+	0.62, 13.0°	0.58(6)	–	–

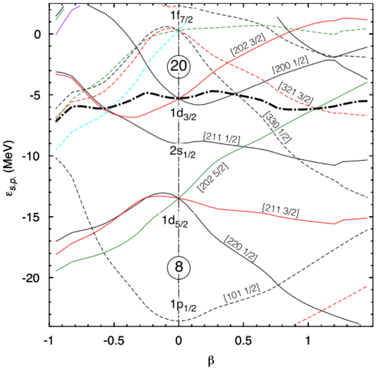

Fig. 2. Single-particle levels of ^{30}Mg for neutrons obtained from the solution of the axially-symmetric HFB equation. The thick dashed lines represent the corresponding Fermi level.

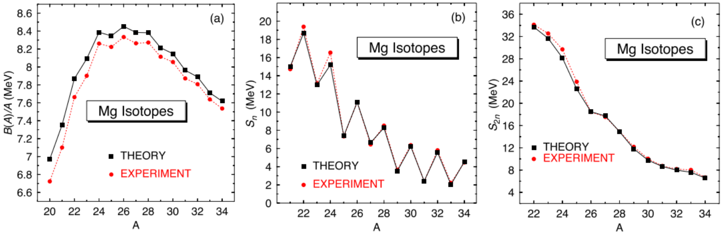

Fig. 3. (a) Binding energy per particle versus de mass number. (b) One-neutron separation energies versus the mass number. (c) Two-neutron separation energies versus the mass number. The experimental values are taken from Ref. [31].

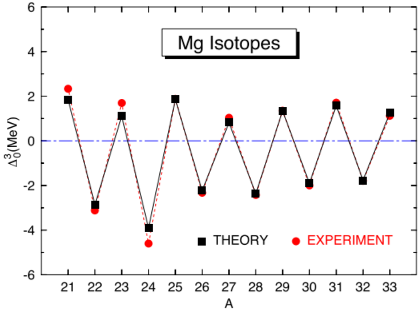

Fig. 4. Odd–even mass differences according to Eq. (18). The experimental data are from Ref. [31].

$$\triangle_0^3(A) = \frac{1}{2}[B(A+1) + B(A-1) - 2B(A)]. \qquad (16)$$

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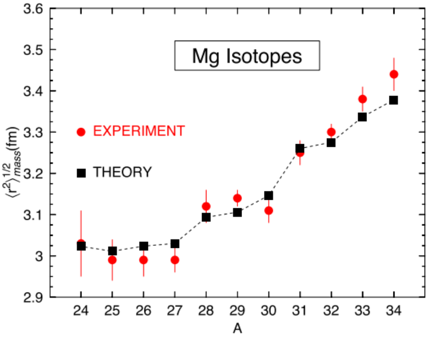

Fig. 5. Radii of the nuclei $^{27-28}$Mg in the PNVAP+PNAMP approach. The experimental data are from Ref. [35].

Results Analysis

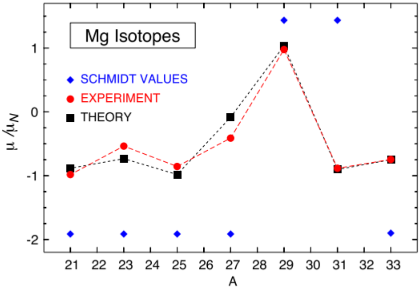

Fig. 6. Magnetic moments of the ground states of the Magnesium isotopes. The experimental results have been taken from the following references: ^{21}Mg [41] ^{21}Mg, [44,45], ^{25}Mg [46], $^{27-31}$Mg [47] and ^{33}Mg [48].

In conclusion, we have presented a novel approach with exact conservation of angular momentum and particle number to describe odd–even nuclei. We have applied this theory to the description of ground-state properties of the Magnesium isotopic chain with the effective Gogny force. The results are in very good agreement with the experimental bulk properties, energy gaps and electromagnetic moments.