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A Brief Introduction to Nuclear Short-Range Correlations

CCWang, Journal Club @2316
10/17/2022

Contents

Short-Range Correlation (SRC)

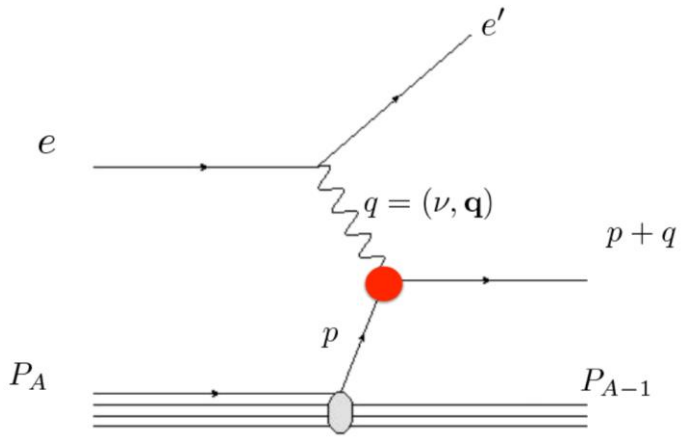
References

RevModPhys89(2017)045002
O. Hen, G A Miller, E. Piasetzky, *et al.*

Nat578(2020)540
A. Schmidt, *et al.*

NatPhys17(2021)306–310
R. Cruz-Torres, *et al.*

1 Experimental aspects on SRC: Kinetic variables/ Bjorken scaling/ hard reactions



$$p^2 = (p+q)^2 = M^2$$

$$2pq = -q^2 = Q^2 > 0$$

$$p = (E_{\text{miss}}, p_{\text{miss}}) \approx (x_E$$

$$x_B = \frac{Q^2}{2M\nu}$$

Deep inelastic scattering (DIS)

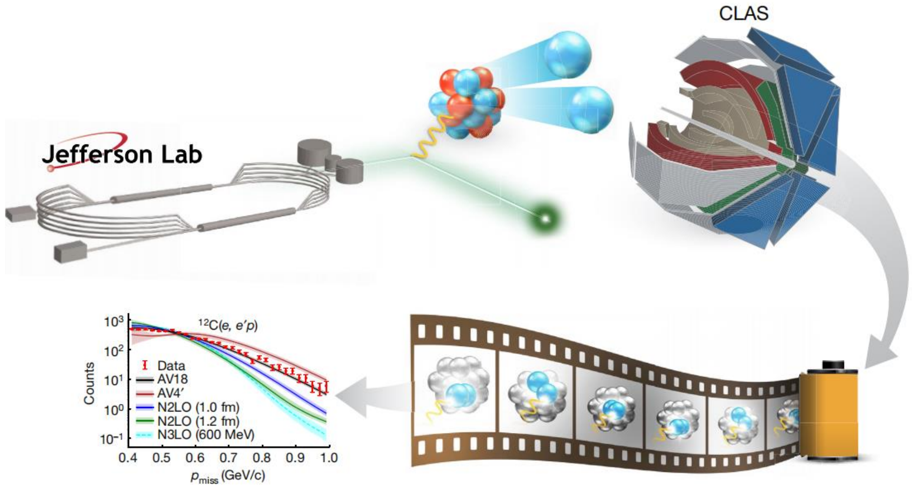
$N\,(e,e')$ Hardrons: $\quad 0.35 < x_B < 0.75$

Inclusive scattering

$A\,(e,e')\,A^*: \quad 1.5 < x_B < 2$

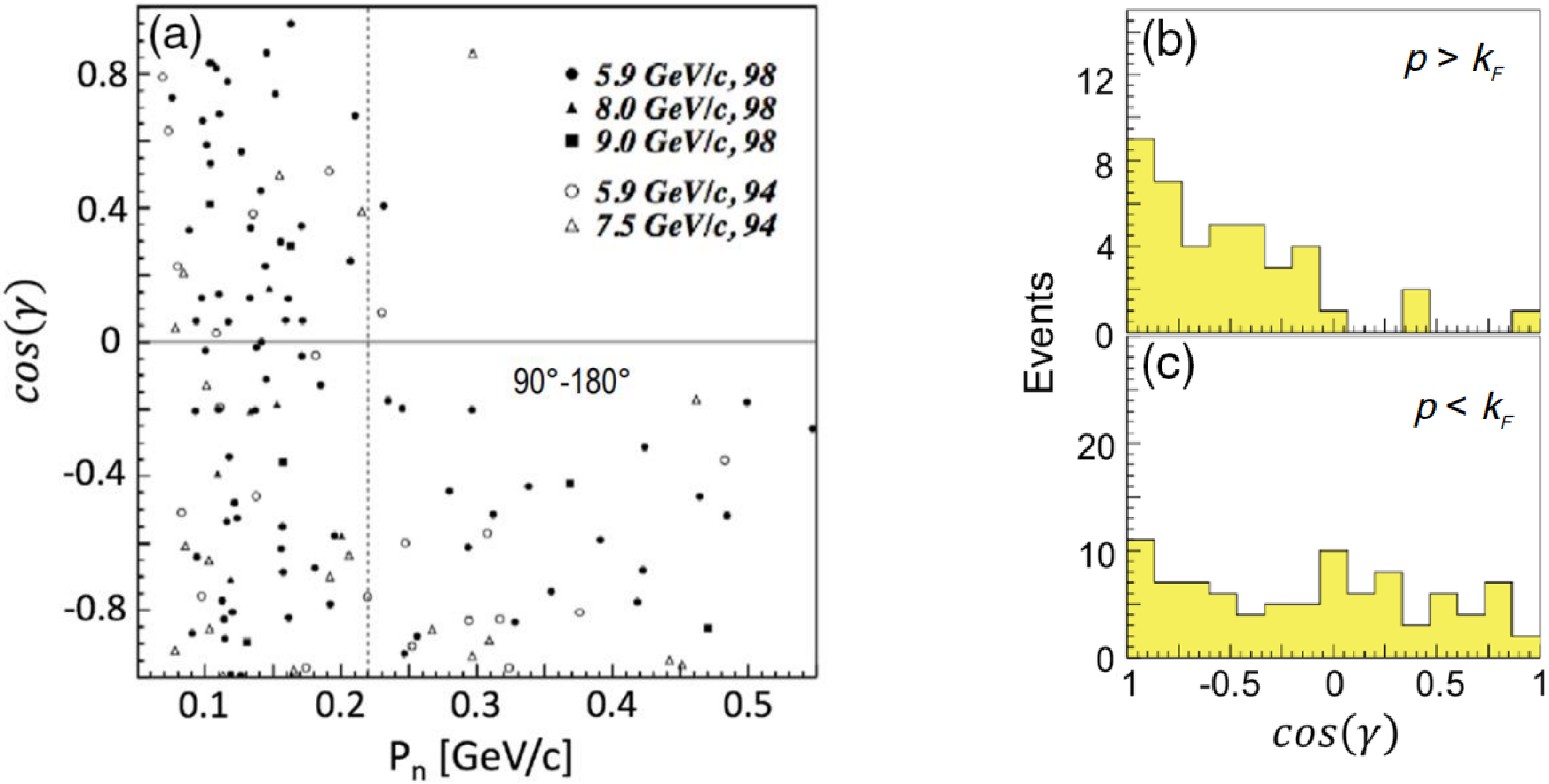
Exclusive scattering

$A\,(e,e'N)\,A-1, \quad A\,(e,e'2N)\,A-2$



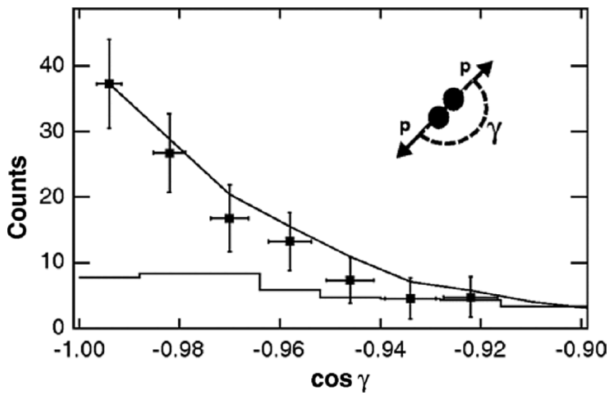
1 Experimental aspects on SRC: exclusive scattering

$^{12}\mathrm{C}(p, 2pn)$ events from BNL

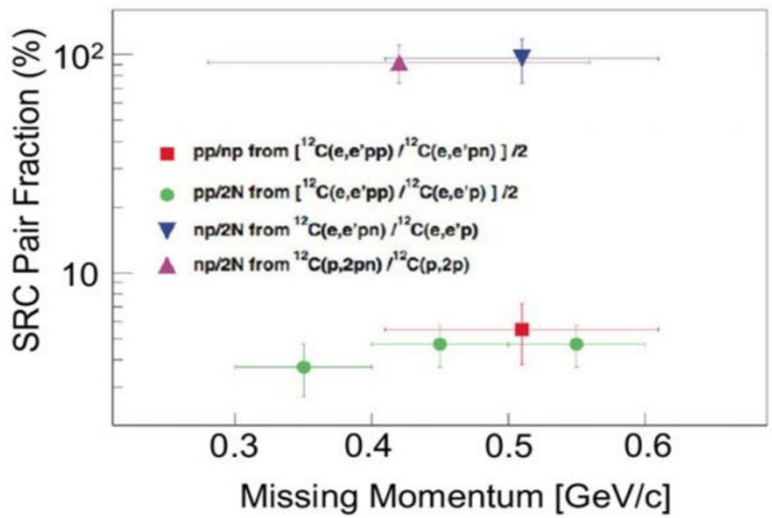


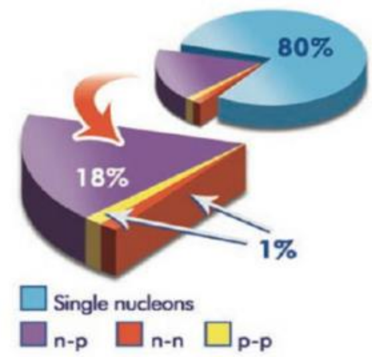
1 Experimental aspects on SRC: exclusive scattering

$^{12}\mathrm{C}(e, e' p p)$ events from JLab



nucleon-nucleon interaction in the measured momentum range. The pie chart on the right illustrates our understanding of the structure of ^{12}C, composed of 80% mean-field nucleons and 20% SRC pairs, where the latter is composed of ~90% np-SRC pairs and 5% pp and nn SRC pairs each. Adapted from Subedi *et al.*, 2008.





1 Experimental aspects on SRC: exclusive scattering

As a result, we can expand the (e, e') cross section into pieces due to electrons scattering from nucleons in two-, three-, and more-nucleon SRCs (Frankfurt and Strikman, 1981, 1988; Frankfurt *et al.*, 1993)

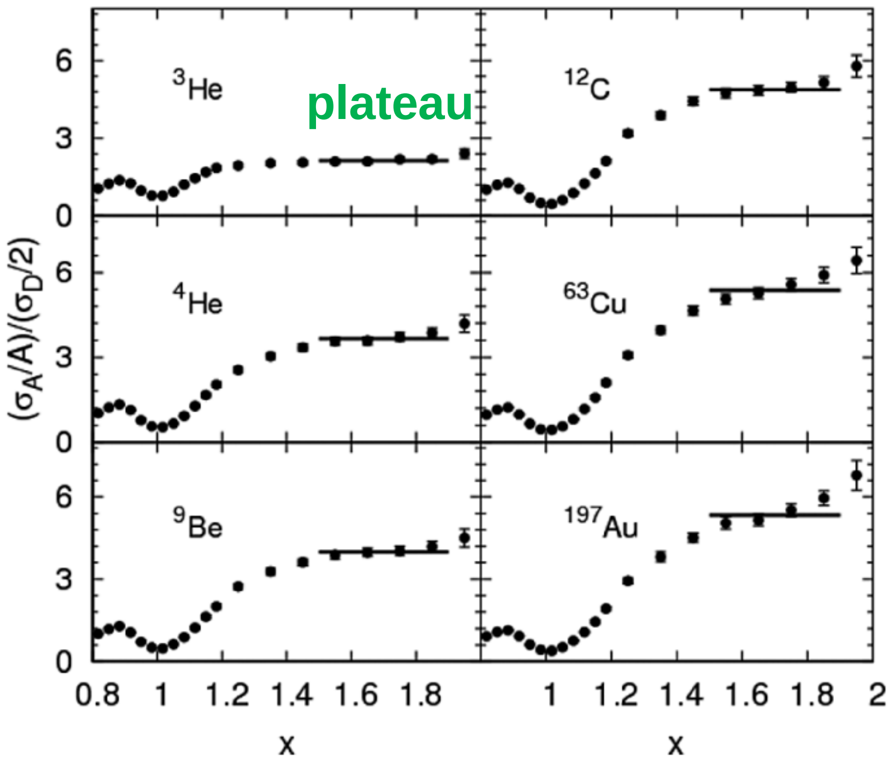
$$\sigma(x_B, Q^2) = \sum_{j=2}^{A} a_j(A)\sigma_j(x_B, Q^2), \qquad (14)$$

where $\sigma_j(x_B, Q^2) = 0$ for $x_B > j$ and the $\{a_j(A)\}$ are proportional to the probability of finding a nucleon in a j-nucleon cluster. This is analogous to treating the nuclear structure in

$$a_2(A) = \frac{2}{A}\frac{\sigma_A(x_B, Q^2)}{\sigma_d(x_B, Q^2)}.$$

This approximation should be valid for $1.5 \lesssim x_B \lesssim 2$.

- **(Plateau) Confirm the validity of expansion (14).**
- **(σ_2 dominance) Higher body correlations $\sigma_{j\geq3}$ are relatively small.**

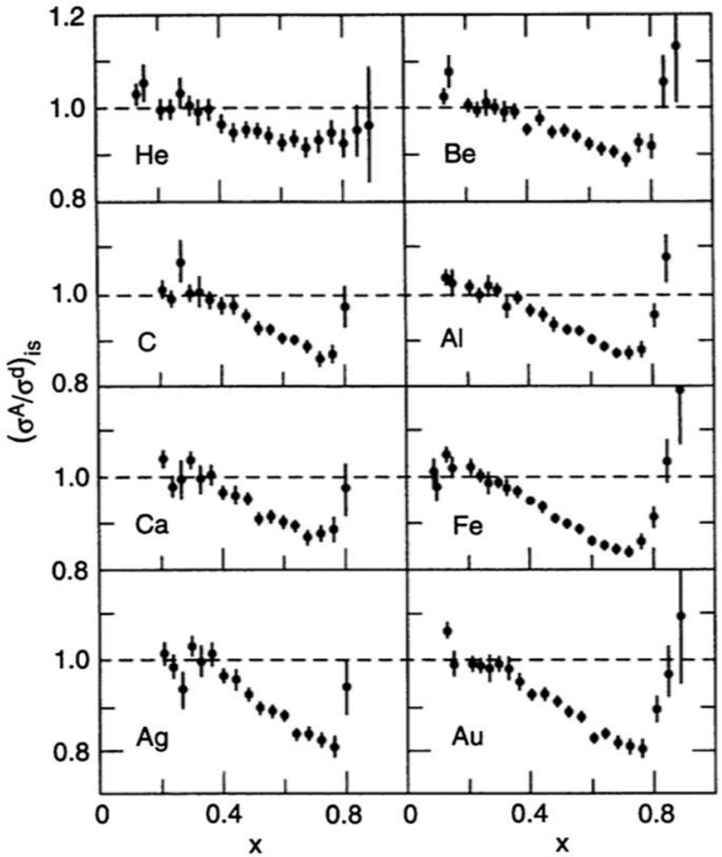


Inclusive per nucleon cross-section ratios of nuclei to deuterium at $Q^2 = 2.7$ GeV^2.

1 Experimental aspects: EMC effect

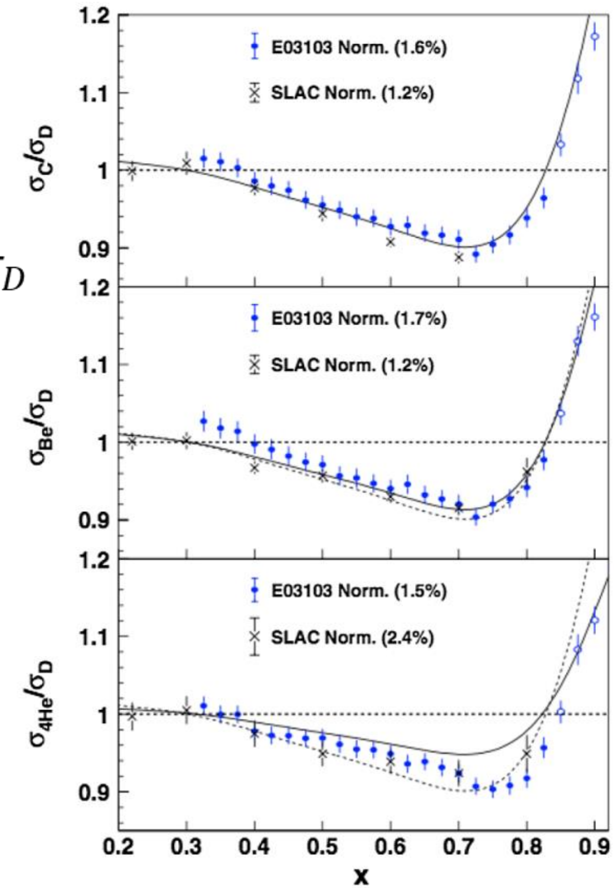
European Muon Collaboration (EMC)

SLAC (Gomez *et al.*, 1994)



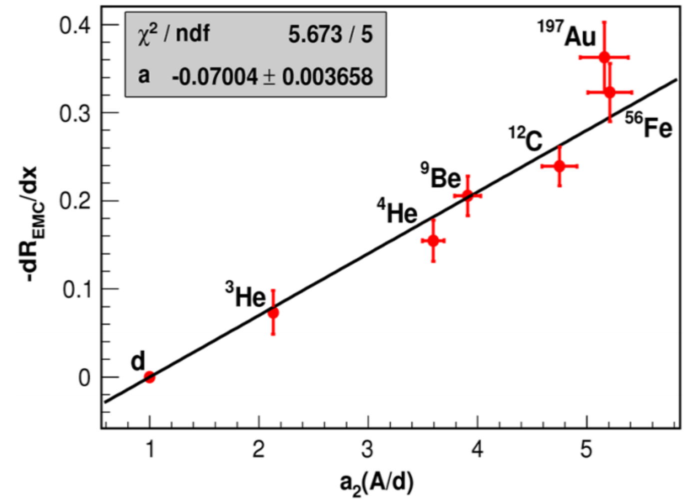
$$R = \mathrm{d}\sigma_A/\mathrm{d}\sigma_D$$

Jefferson Lab (Seely *et al.*, 2009).



$\mathrm{d}R/\mathrm{d}x$ measures the EMC effect.

1 SRC & EMC effect: correlation



Nucleus	Frankfurt *et al.* (1993) $a_2(A)$	Egiyan *et al.* (2006) $a_2(A)$	Fomin *et al.* (2012) Excluding the c.m. motion correction	Weinstein *et al.* (2011) EMC-SRC prediction $a_2(A)$	Weinstein *et al.* (2011) EMC slope (dR_{EMC}/dx)
Column No.	2	3	4	5	6
^{3}He	1.7 ± 0.3	1.97 ± 0.10	2.13 ± 0.04		-0.070 ± 0.029
^{4}He	3.3 ± 0.5	3.80 ± 0.34	3.60 ± 0.10		-0.197 ± 0.026
^{9}Be			3.91 ± 0.12	4.08 ± 0.60	-0.243 ± 0.023
^{12}C	5.0 ± 0.5	4.75 ± 0.41	4.75 ± 0.16		-0.292 ± 0.023
^{56}Fe(^{63}Cu)	5.2 ± 0.9	5.58 ± 0.45	5.21 ± 0.20		-0.388 ± 0.032
^{197}Au	4.8 ± 0.7		5.16 ± 0.22	6.19 ± 0.65	-0.409 ± 0.039
EMC-SRC slope		0.079 ± 0.006	0.084 ± 0.004		
$\frac{\sigma(n+p)}{\sigma_d}\vert_{x_B=0.7}$		1.032 ± 0.004	1.034 ± 0.004		
χ^2/ndf		$0.7688/3$	$4.895/5$		

2 Theoretical aspects: Variational Monte Carlo (VMC) methods

PhysRevLett98(2007)132501 R. Schiavilla, *et al.*

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NatPhys17(2021)306–310 R. Cruz-Torres, *et al.*

...

Hamiltonian Model: Argonne *v*18 (2N)+ Urbana/Illinois (3N)

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij})\, \mathcal{O}_{ij}^p$$

$$V_{ijk} = V_{ijk}^{2\pi,A} + V_{ijk}^{2\pi,C} + V_{ijk}^R.$$

Few/Many-Body methods: VMC

$$|\Psi_V\rangle = \left(1 + \sum_{i<j<k} U_{ijk}\right)\left[\mathcal{S}\prod_{i<j}\left(1 + U_{ij}^{2-6}\right)\right]\left[1 + \sum_{i<j} U_{ij}^{7-8}\right]|\Psi_J\rangle$$

$$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij})\right]\mathcal{A}\,|\Phi\rangle$$

$$E_V = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \geqslant E_0$$

2 Theoretical aspects: Calculation of SRG, MC integration

R-space

On-body density: $\rho(\mathbf{x}) = \frac{1}{A}\left\langle \Psi \right| \sum_{i=1}^{A} \delta(\mathbf{x}-\mathbf{x}_i) \left| \Psi \right\rangle$

Two-body density:

$$\rho^{(2)}(\mathbf{x},\mathbf{y}) = \frac{1}{A(A-1)} \langle \Psi | \sum_{i \neq j} \delta(\mathbf{x}-\mathbf{x}_i)\delta(\mathbf{y}-\mathbf{y}_j) | \Psi \rangle .$$

Two-body pair with relative distance r:

$$\rho_{2,1}(r) \equiv \frac{1}{4\pi r^2 A} \langle \Psi | \sum_{i \neq j} \delta(r - |\mathbf{r}_i - \mathbf{r}_j|) \Psi \rangle = \int d^3R \rho_2(\mathbf{R}+\mathbf{r}/2, \mathbf{R}-\mathbf{r}/2),$$

Probability for a nucleon having momentum k: $n(\mathbf{k}) = \frac{1}{A} \langle \Psi | \sum_{i=1}^{A} \delta(\mathbf{k}-\mathbf{k}_i) | \Psi \rangle .$

total momentum of K and a relative momentum κ:

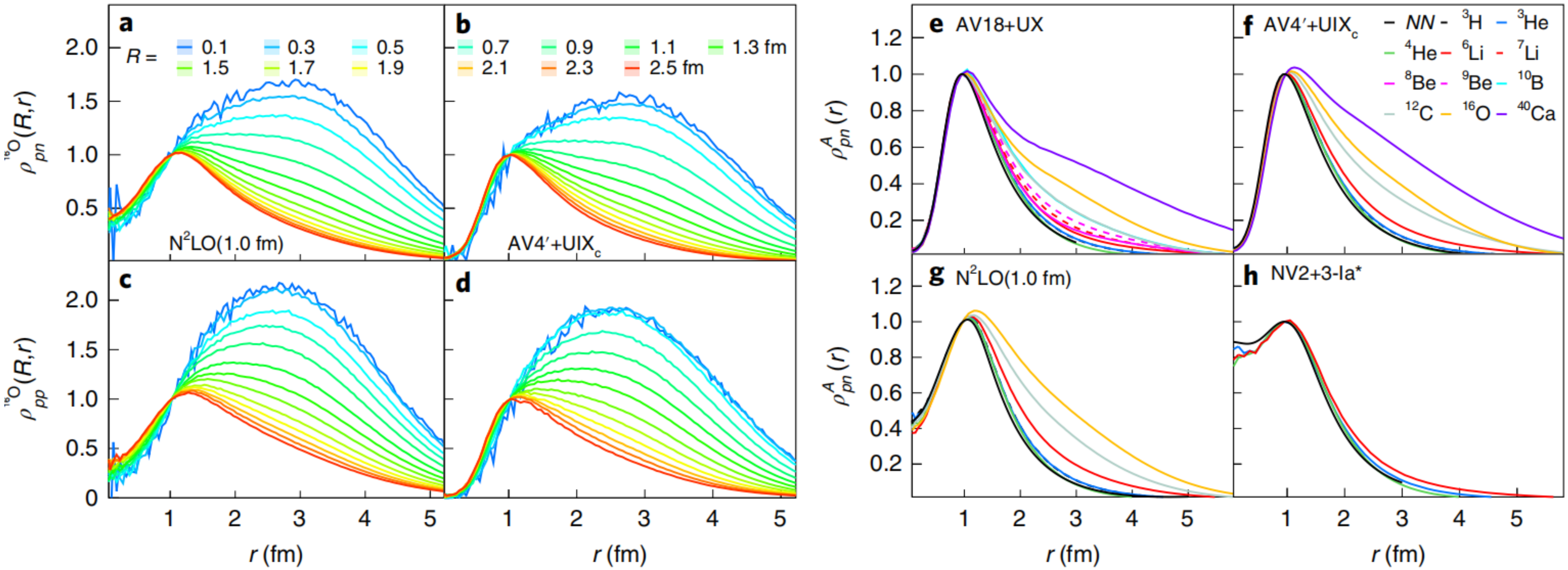
$$n_2(\mathbf{K},\boldsymbol{\kappa}) = \frac{1}{A(A-1)} \left\langle \Psi \right| \sum_{i \neq j} \delta(\mathbf{K}/2 + \boldsymbol{\kappa} - \mathbf{k}_i) \quad \delta(\mathbf{K}/2 - \boldsymbol{\kappa} - \mathbf{k}_j) \left| \Psi \right\rangle$$

$$n_{2,1}(\boldsymbol{\kappa}) \equiv \int d^3K n_2(\mathbf{K},\boldsymbol{\kappa}) = \frac{2}{A(A-1)} \left\langle \Psi \right| \sum_{i \neq j} \delta(\mathbf{k}_i - \mathbf{k}_j - 2\boldsymbol{\kappa}) \left| \Psi \right\rangle$$

Q-space

2 Theoretical aspects: R-space results

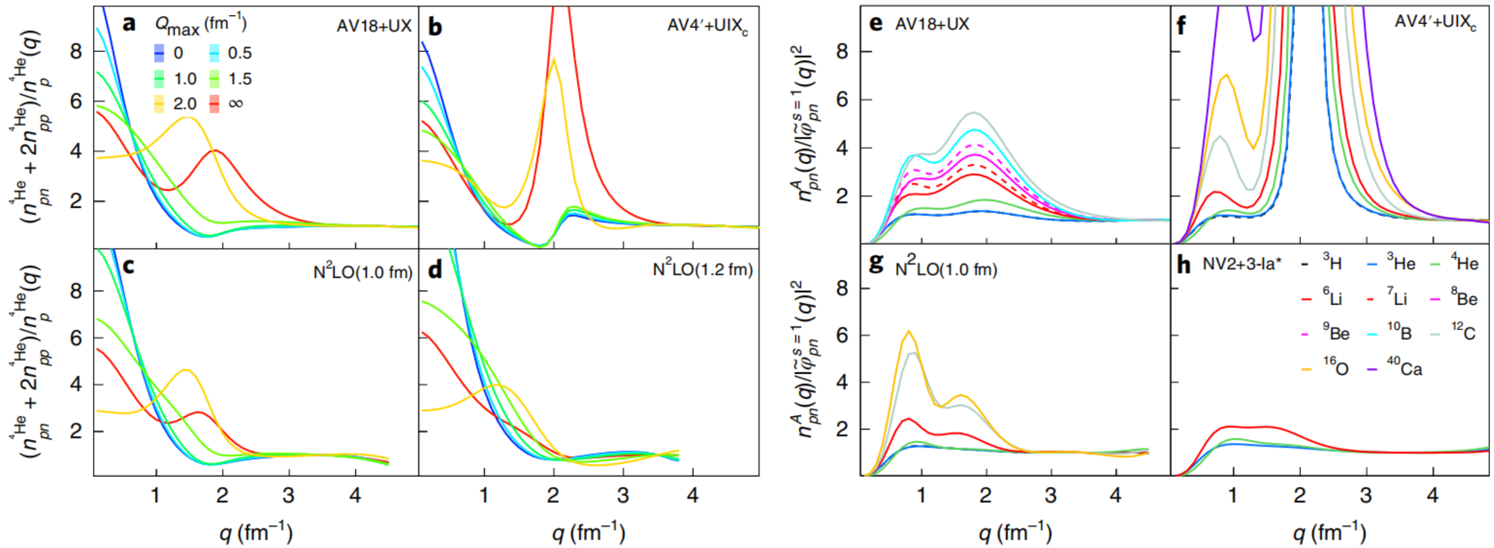
$$\rho^A_{NN}(r) = \int d\mathbf{R}\, \rho^A_{NN}(\mathbf{R}, \mathbf{r})$$



- (a-d) Similarities of the different distributions at $r \leq 1$ for all values of R, manifesting the existence of short-distance factorization.
- (e-f) Scale & scheme (MODEL) dependence on predictions of two-body density & wave functions (WF), BUT universal factorization of different short-distance WFs.

2 Theoretical aspects: Q-space results

$$[\int_0^{Q_{max}} (n_{np}^{^4He}(Q,q) + 2n_{pp}^{^4He}(Q,q))dQ]/n_p^{^4He}(q)$$



Generalized Contact Formalism (GCF)

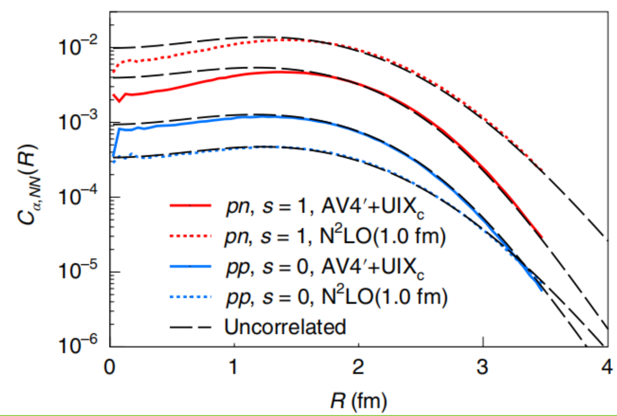
$$\rho_{\alpha,NN}^A(R,r) = C_{\alpha,NN}^A(R) \times |\varphi_{NN}^\alpha(r)|^2,$$

$$n_{\alpha,NN}^A(Q,q) = \tilde{C}_{\alpha,NN}^A(Q) \times |\tilde{\varphi}_{NN}^\alpha(q)|^2,$$

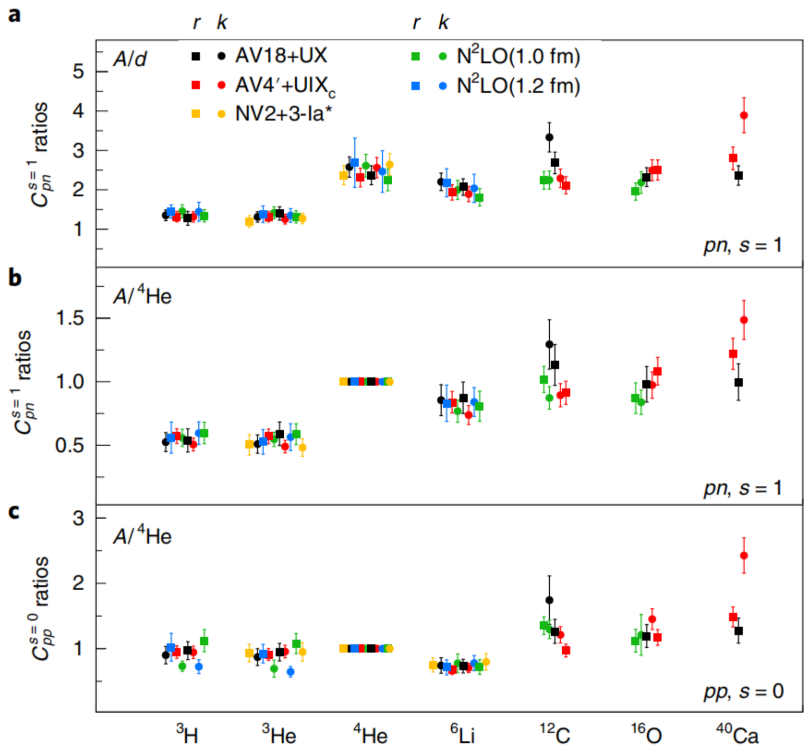
$$C_{\alpha,NN}^A \equiv \int d\mathbf{R}\ C_{\alpha,NN}^A(R),$$

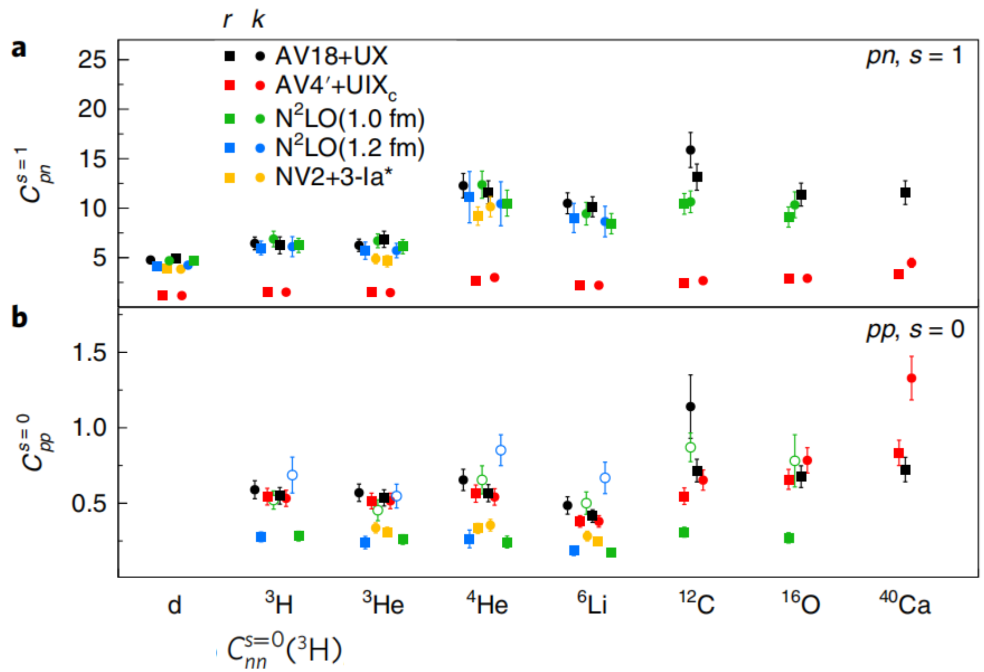
$$\tilde{C}_{\alpha,NN}^A \equiv \frac{1}{(2\pi)^3} \int d\mathbf{Q}\ \tilde{C}_{\alpha,NN}^A(Q),$$

number of *NN* SRC pairs in nucleus *A*



2 Theoretical aspects: contact terms





- **(Left) Pair ratio in various nuclei.**
- **(Right) Manifestation of tensor force effect**
  (contacts are divided by *A*/2 and multiplied by 100).

2 Theoretical aspects: SRC & core of nuclear interaction

At the high-Q^2 kinematics of our measurement, the differential A(e, $e'p$) nucleon knockout cross-sections can be approximately factorized as[14,21]

$$\frac{d^6\sigma}{d\Omega_{\mathbf{k}'}\, d\epsilon'_k\, d\Omega_{\mathbf{p}_N}\, d\epsilon_N} = p_N \epsilon_N \sigma_{ep} S(\mathbf{p}_i, \epsilon_i) \qquad (1)$$

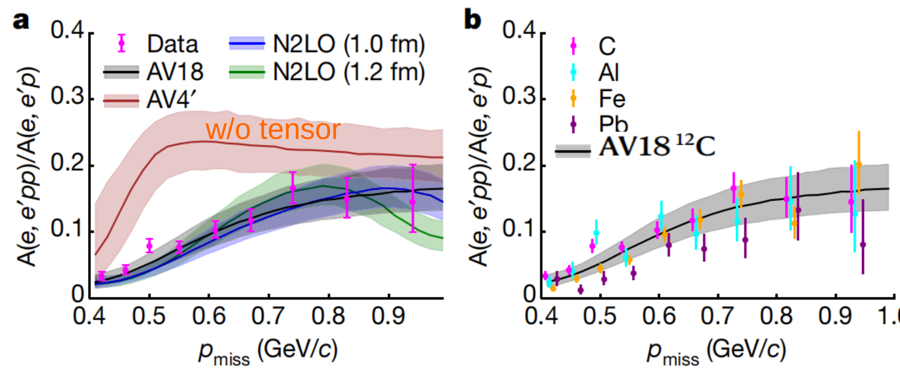
where $\Omega_{\mathbf{k}'}$ and $\Omega_{\mathbf{p}_N}$ are the scattered electron and knockout proton solid angles, $\mathbf{k}'$ ($\mathbf{p}_N$) and ϵ'_k (ϵ_N) are the final electron (proton) momentum and energy, σ_{ep} is the off-shell electron–proton cross-section[21] and $S(\mathbf{p}_i, \epsilon_i)$ is the nuclear spectral function that defines the probability for finding a nucleon in the nucleus with[22] momentum $\mathbf{p}_i$ and energy ϵ_i.

$$S_p(\mathbf{p}, \epsilon) = \frac{1}{A}\sum_i \langle \Psi | \delta_p(\mathbf{p}_i - \mathbf{p})\, \delta_p(\epsilon_i - \epsilon) | \Psi \rangle$$

formalism (GCF)[22–25] which assumes that at very high momenta, the nuclear wavefunction can be described as consisting of an SRC pair and a residual $A-2$ system. The abundance of SRC pairs is given by nuclear contact terms extracted from ab initio many-body calculations of pair momentum distributions[24,25].

Therefore, in the GCF, the high-momentum proton spectral function of equation (1) is approximated by a sum over pp and pn SRC pairs, which enables the calculation of the cross-sections of (e, $e'p$) and (e, $e'pp$) using different nuclear interaction models as input[13,22].

$$|\Psi\rangle = |\Psi_{A-2}\rangle |\rho_{pN}\rangle$$

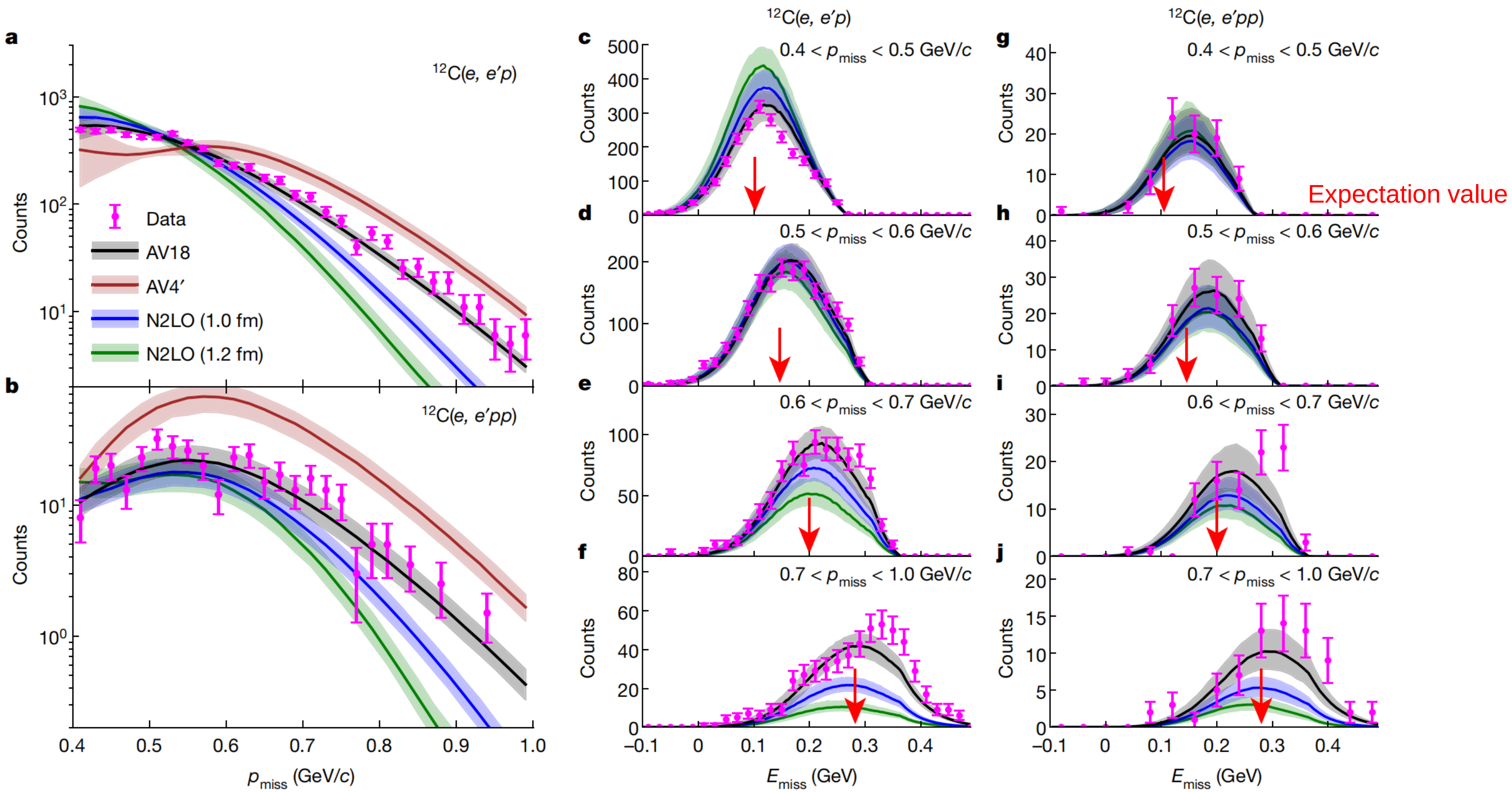


- (a) measured & calculated (e, $e'pp$)/(e, $e'p$) event ratio as a function of p_{miss} for carbon, aluminium, iron and lead.

At high momenta all calculations predict a pp SRC pair fraction of about 1/3 (Extended Data Fig. 6c), which is equal to the scalar limit

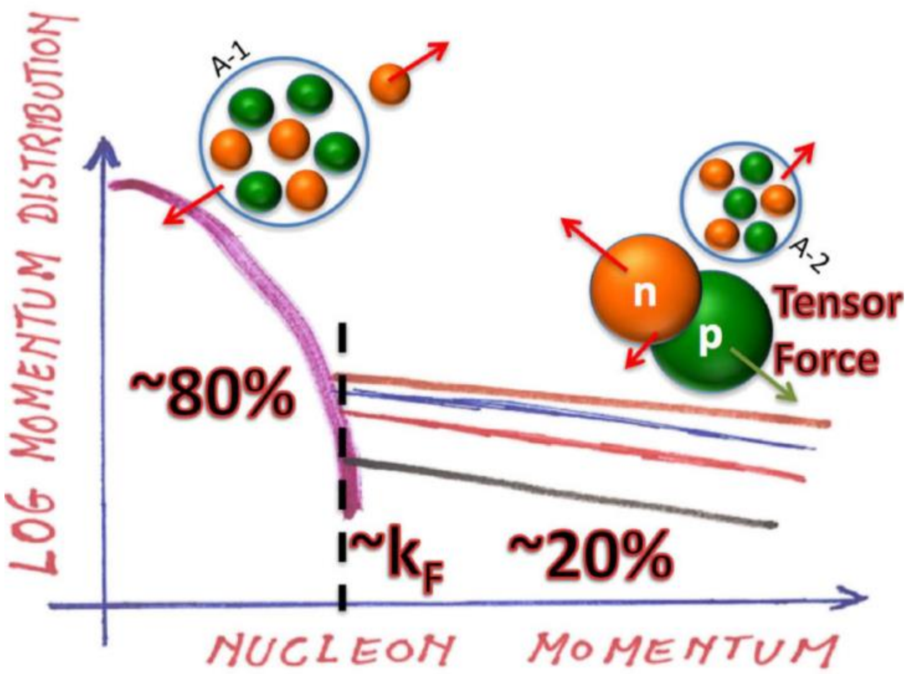
- (b) Experimental results vs. theoretical calculation of ^{12}C (nucleus independent).

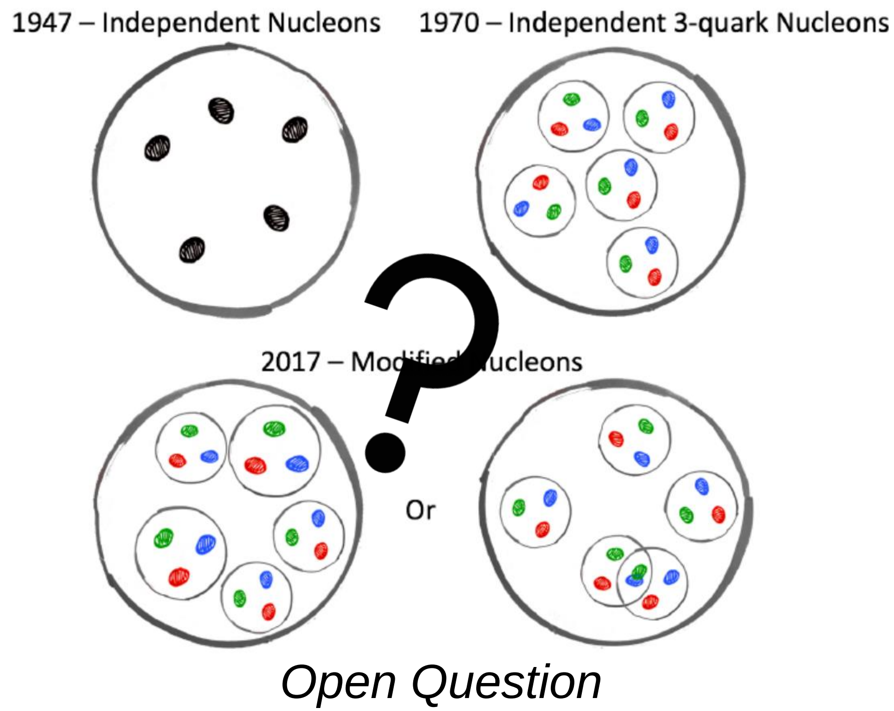
2 Theoretical aspects: SRC & core of nuclear interaction, cont'd

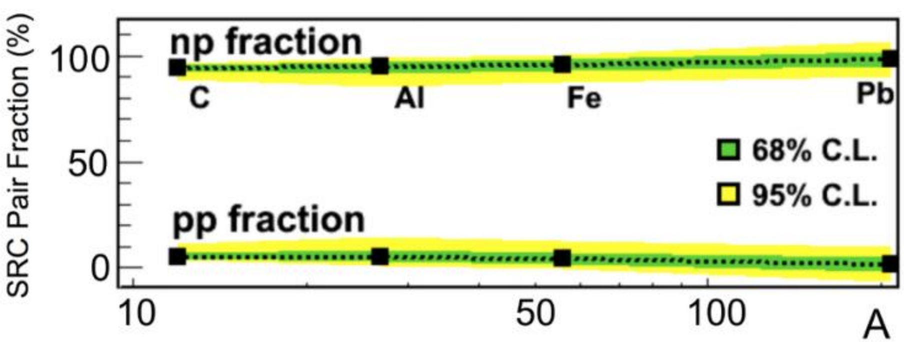


- As the relative momentum between two nucleons increases, a transition from a spin-dependent tensor force to a predominantly spin-independent scalar force.
- The results provide strong support for the use of point-like nucleons with effective interactions for modelling atomic nuclei and dense astrophysical systems such as neutron stars, the outer core of which exceeds the nuclear saturation density under current models.

3 Summary: the physical pictures







Thank you for your attentions!