Schiff moment

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symmetry violation

An important conservation formula

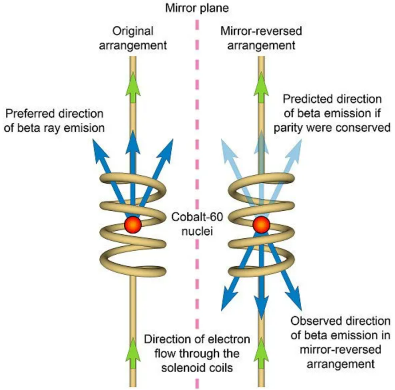
$$SHS^{-1} = H \tag{1.1}$$

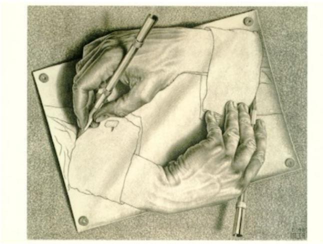
where S, H are the symmetry operator and Hamiltonian,respectively, the former such as P, T so on. And of course you could argue that if some operator is commutative to the Hamiltonian then that quantity is conserved, that the Hamiltonian has some kind of symmetry.One of the most typical examples is parity conservation, but this case is not true for weak interactions that is parity breaking. So

$$SHS^{-1} \neq H \tag{1.2}$$

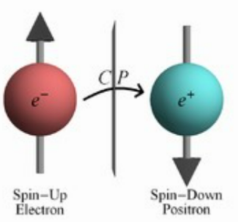
symmetry violation

Through the experiment of ^{60}Co's β decay to compare the distribution of electrons and photons with different spins, if the proportion in the same spin direction is the same, it can be concluded that the parity of weak interaction is conserved, but if there is a counterexample, Yang Zhenning and Li Zhengdao's hypothesis is correct.



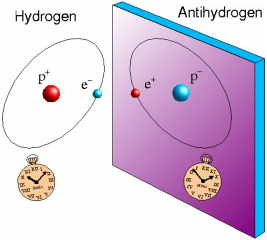


SHORTLY after the suggestion by Lee and Yang[1] that parity (P) is not conserved in weak interactions, Landau[2] pointed out that invariance under the combined operation CP of charge conjugation (C) and parity is needed to rule out the existence of static electric dipole moments of elementary particles. While there is ample evidence that the weak interactions are not invariant under C and P separately, they may be invariant under CP.[3] The CPT theorem would then imply invariance under time reversal (T). However,



$$CPT = 1 \tag{1.3}$$

According to the conservation of CPT, hydrogen atoms obey the same laws of physics as antihydrogen atoms in a mirror that travel backwards in time.



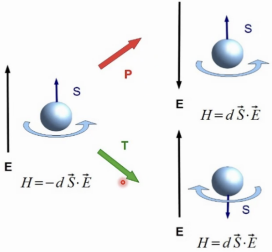
symmetry violation

Electric dipole moment (EDM)

- EDM measures the polarity of a charged system, $\vec{d} = \sum_i q_i \vec{r}_i$
- For a hadron or any elementary particle at rest, $\vec{d} = d\,\frac{\vec{S}}{|\vec{S}|}$
- Hamiltonian for a dipole interacting with an electric field

$$\mathcal{H}_{\rm edm} = -\vec{d}\cdot\vec{E} = -d\,\frac{\vec{S}\cdot\vec{E}}{|\vec{S}|}$$

- EDM leads to P and T(CP) odd interactions



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abstract

The possibility of measuring a very small nuclear electric dipole moment is explored by calculating the interaction of this moment with an external electric field. It is shown that for a quantum system of point, charged, electric dipoles in an external electrostatic potential of arbitrary form, there is complete shielding; i.e.,there is no term in the interaction energy that is of first order in the electric dipole moments, regardless of the magnitude of the external potential. This is true even if the particles are of finite size, provided that the charge and dipole moment of each have the same spatial distribution. Relativistic and second-order effects are uninterestingly small. There is, however, a first-order interaction if the charge and moment distributions are diferent, and also for a point electric dipole if it also carries a magnetic dipole moment. Explicit calculations of both effects are given for hydrogen and helium atoms. It is found that the effective electric field at a He^3 nucleus arising from the magnetic dipole effect is about a hundred times that arising from the finite size effect, and is roughly 10^{-7} times the external electric field.

Introduction

- However, observations on the correlation between the neutron spin vector and the proton and electron momentum vectors in the decay of polarized neutrons leave open the possibility of an appreciable breakdown of T invariance, and other kinds of experiments do not appear to restrict this possibility significantly.
- Thus, it is worthwhile to consider attempting the measurement of a nuclear electric dipole moment, or indeed of any "odd" nuclear moment (magnetic monopole or quadrupole, electric octupole, etc.).
- Measurement of higher "odd" moments is subject to the following general difficulty. The environmental electric field must be made exceedingly small in comparison with the magnetic field of the same symmetry, and both this electric field and the electric quadrupole moment must be known with great accuracy.

- The measurement of an electric dipole moment in the presence of a much larger magnetic dipole moment is relatively favorable.Smith, Purcell, and Ramsey' attempted to measure the change in the precession frequency of neutrons in a weak uniform magnetic field when a strong uniform electric field was superposed parallel to the magnetic field.
- They found that if the neutron electric dipole moment is written as $e = eD$, where e is the electronic charge, then $D < 10^{-20} cm$
- The remainder of this paper is devoted to a discussion of the extent to which a nuclear electric dipole moment can be made to interact with an externally applied electric field.

A simple classical argument also shows that it is not helpful to use a time-dependent electric field. The equation of motion of the angular momentum vector $\mathbf{J}$ of a classical electric dipole $\boldsymbol{\mu}$ in an electric field $\mathbf{E}$ is $d\mathbf{J}/dt=\boldsymbol{\mu}\times\mathbf{E}$. Now we can put $\mu=eD$, where D is probably less than 5×10^{-20} cm, so that for any reasonable value of E, the precession period will be very long. Thus we can assume that $\boldsymbol{\mu}$ is nearly fixed in space, say along the z axis, and calculate the rotation of $\boldsymbol{\mu}$, and hence of the parallel vector $\mathbf{J}$, about the x and y axes. The angular velocity about the x axis is $d\theta_x/dt=-\mu E_x/J$, and there is a similar relation for the rotation about the y axis. The x component of the acceleration of the nucleus, of charge Ze and mass AM, is given by $dv_x/dt=ZeE_x/AM$. Thus $\Delta\theta_x=-\mu AM\Delta v_x/JZe=-(\gamma A/IZ)(\Delta v_x/c)$, where we have put $J=I\hbar$ and expressed D as a multiple γ of

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The nonrelativistic Hamiltonian for a system of particle of finite size, with mass m_i, charge e_i,electric dipole moment $\mathbf{d_i}$,and center-of-mass coordinate $\mathbf{r_i}$ in an external electric potential $\phi(\mathbf{r})$,may can be written:

$$H = T + V_0 + V + U + W \tag{4.1}$$

where

$$T = -\sum_i \frac{\hbar^2}{2m_i}\vec{\nabla}^2 \tag{4.2}$$

$$V_0 = \sum_i \sum_j {}_{i>j} e_i e_j \int\int \frac{\rho_{ic}(\mathbf{r})\rho_{jc}(\mathbf{r}')}{|\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}'|} d^3 r \tag{4.3}$$

$$V = \sum_i e_i \int \rho_{ic}(\mathbf{r})\phi(\mathbf{r}_i + \mathbf{r}) d^3 r \tag{4.4}$$

$$U = \sum\sum_{i\neq j} e_i \mathbf{d}_j \cdot \int\int \frac{\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}'}{|\mathbf{r}_i - \mathbf{r}_j + \mathbf{r} - \mathbf{r}'|^{-3}} \rho_{ic}(\mathbf{r})\rho_{jM}(\mathbf{r}') d^3 r d^3 r' \tag{4.5}$$

$$W = \sum_i \mathbf{d}_i \cdot \nabla_i \int \rho_{iM}(\mathbf{r})\phi(\mathbf{r}_i + \mathbf{r}) d^3 r \tag{4.6}$$

Direct dipole-dipole interaction terms, of order $\vec{d}_i, \vec{d}_j$, have been neglected. The charge and dipole moment distribution functions, ρ_{ic} and ρ_{iM}, are normalized to unit volume integral.

We define the infinitesimal displace operator

$$Q = \frac{1}{e_i \hbar} \sum_i \mathbf{d}_i \cdot \mathbf{p}_i \tag{4.7}$$

where $\mathbf{p}_i$ is the momentum operator for the *ith* particle.It is easily seen that Q commutes with T, and that

$$i[Q, V_0] = U', \qquad i[Q, V] = W' \tag{4.8}$$

where U' and W'are the same as U and Wexcept that ρ_{iM} is replaced by ρ_{iC}. Thus, if we call the Hamiltonian in the absence of dipole moments

$$H_0 = T + V_0 + V \tag{4.9}$$

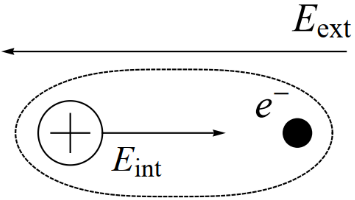
and

$$H = H_0 + i[Q, H] + \Delta U + \Delta W, \qquad \Delta U = U' - U, \Delta W = W' - W$$

In the remainder of this section we shall assume that $\rho_{iC} = \rho_{iM}$, so that $\Delta U = \Delta W = 0$. It shows that H is the same as Ho except for the displacement of each particle by the vector $\frac{\vec{d}_i}{e_i}$, provided that these vectors are regarded as being infinitesimal. This is in agreement with the classical view of a charged dipole.

– for a neutral system made up of point charged particles which interact electrostatically with each other and with an arbitrary external field, the shielding is complete (Schiff, 1963):

$$\langle \mathbf{E} \rangle|_{on\ any\ q} = \langle \mathbf{E}_{ext} + \mathbf{E}_{int} \rangle|_{on\ any\ q} \equiv 0,$$



There is no any way to “feel” nuclear EDM since:

$$\langle -\mathbf{d_N} (\mathbf{E}_{ext} + \mathbf{E}_{int}) \rangle = 0$$

where $\delta\rho$ is the correction to the charge density necessitated by the P- or T-odd interaction. If, however, a neutral atom or molecule is regarded as a system of pointlike particles with Coulomb interaction then, even though the nucleus has an EDM, the total dipole moment of the system is zero in accordance with Schiff's well known theorem.[26] (A detailed analysis of a number of problems connected with this theorem can be found, e.g., in Khriplovich's book[27].) It was noted in Ref. 26, however, that this hindrance is lifted, in particular, when the finite dimensions of the nucleus are taken into account. It was shown subsequently[28] that it is precisely this effect which is the main cause of violation of Schiff's theorem in heavy atoms and molecules. The Schiff hindrance reduces to the fact that the P- and T-odd potentials of the nucleus must be written in the form

$$\delta\varphi(\mathbf{R}) = e\int\frac{\delta\rho(\mathbf{r})\,d^3r}{|\mathbf{R}-\mathbf{r}|} + \frac{1}{Z}(\mathbf{d}\nabla)\int\frac{\rho_0(r)\,d^3r}{|\mathbf{R}-\mathbf{r}|}, \qquad (22)$$

It can also be written in terms of the finite displacement operator e^{iQ} by subtracting out the higher order terms

$$H = e^{iQ} H_0 e^{-iQ} + \frac{1}{2}[Q,[Q,H]] + \cdots \tag{4.10}$$

As are the neglected dipole-dipole terms, the first order in the $\vec{d}_i$, the eigenfunctions $u_N$of H_0, which satisfy the Schrodinger equation

$$H_0 u_n = E_n u_n \tag{4.11}$$

determine in a simple way the eigenfunctions $e^{iQ} u_n$of H, which satisfy the Schrodinger equation

$$H \ e^{iQ} u_n = E_n e^{iQ} u_n \tag{4.12}$$

We conclude that this is also true of the eigenvalues of the above equation, so that there is no interaction energy of first order in the dipole moments. This result depends on the assumption that the charge and moment distribution functions are the same, but is valid for an external potential of arbitrary form and magnitude.

SECOND-ORDER INTERACTION FOR A POINT ELECTRIC DIPOLE

For simplicity, we restrict ourselves in this section to a signal point dipole of mass m_0, charge e_0, electric dipole moment $\mathbf{d} = d\vec{\sigma}$, and a number of point charge describe by m_i, e_i, r_i. We choose a dipole of spin $\frac{1}{2}$ for definiteness,so that the components of $\tilde{\sigma}$ are the Pauli spin matrices. Proof:

$$\frac{1}{2}[Q,[Q,V]] = \frac{d^2}{e_0\hbar}\vec{\sigma}\cdot(\vec{E}\times\vec{p}) \tag{4.13}$$

Where $V = e_0\psi(\vec{r})$, so that

$$[Q,V] = \frac{1}{\hbar}[\vec{d}\cdot\vec{p},\psi(\vec{r})] = \frac{1}{\hbar}\,\vec{d}\cdot(-i\hbar\vec{\nabla}\psi(\vec{r})) = -i\,\vec{d}\cdot\vec{\nabla}\psi(\vec{r}) = -iW \tag{4.14}$$

and

$$[Q,[Q,V]] = \frac{-i}{e_0\hbar}\,[\vec{d}\cdot\vec{p},\vec{d}\cdot\vec{\nabla}\psi(\vec{r})] = \frac{1}{e_0\hbar^2}\,[\vec{d}\cdot\vec{p},\vec{d}\cdot\vec{p}\psi(\vec{r})] \tag{4.15}$$

We define the relationship, $E = -\vec{\nabla} \cdot \psi = -i \frac{\vec{p} \cdot \psi}{\hbar}$. So

$$\frac{1}{e_0 \hbar^2} [\vec{d} \cdot \vec{p}, \vec{d} \cdot \vec{p} \psi(\vec{r})] = \frac{id^2}{e_0 \hbar} [\vec{\sigma} \cdot \vec{p}, \vec{\sigma} \cdot \vec{E}]$$

$$= \frac{id^2}{e_0 \hbar} [(\vec{\sigma} \cdot \vec{p}) \cdot (\vec{\sigma} \cdot \vec{E}) - (\vec{\sigma} \cdot \vec{E}) \cdot (\vec{\sigma} \cdot \vec{p})] = 2 \frac{d^2}{e_0 \hbar} \vec{\sigma} \cdot (\vec{E} \times \vec{p}) \quad (4.16)$$

where we use $(\vec{\sigma} \cdot \vec{A}) \cdot (\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} - i \vec{\sigma} \cdot (\vec{A} \times \vec{B})$.

FIRST-ORDER INTERACTION FOR ELECTRIC DIPOLES OF FINITE SIZE

We have

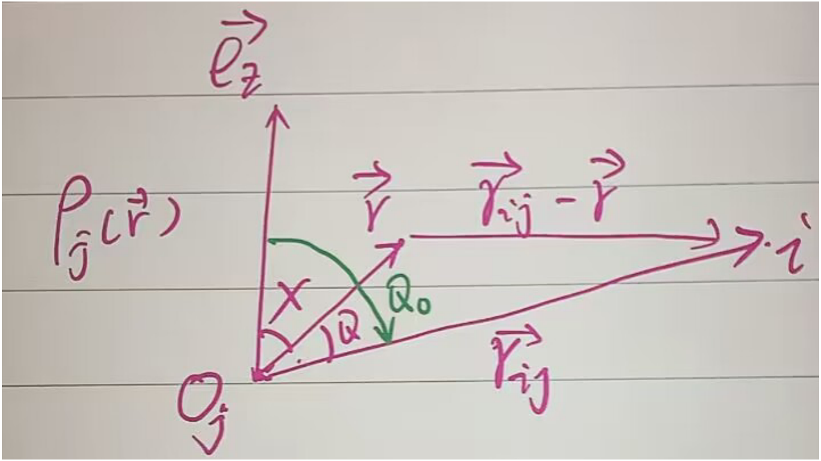
$$H = H_0 + i[Q, H] + \Delta U + \Delta W \tag{4.17}$$

We no longer assume that $\rho_{iC(\vec{r})} = \rho_{iM(\vec{r})}$, but define the difference distribution function

$$\rho_i(\vec{r}) = \rho_{iC}(\vec{r}) - \rho_{iM}(\vec{r}) \tag{4.18}$$

Since $\rho_{iC}(\vec{r})$and $\rho_{iM}(\vec{r})$ are normalized, the volume integral of ρ_i is zero. It is sufficient for the experimental situation to regard the electric field as uniform, in which case $\Delta W = 0$. Further, the particles may be assumed to be small in comparison with their mean separations, so that we need calculate only the leading term in a power series of the ratio of size to separation. Thus in the expression for ΔU,

$$\Delta U = -\sum\sum_{i \neq j} e_i \vec{d}_j \cdot \int (r_{ij} - r)\rho_j(r) \times |r_{ij} - r|^{-3} d^3 r, \quad r_{ij} = r_i - r_j$$



where $\rho_j(\vec{r})$ is the dipole moment distribution function and can be normalized to unit volume integral, $\vec{z}$ represent a symmetry axis for $\rho(\vec{r})$, O_j is the center of dipole moment, i represent charge.

Derivation:

$$\Delta U = -\sum \sum_{i \neq j} e_i \cdot \vec{d}_j \cdot \int \frac{\vec{r}_{ij} - \vec{r}}{|\vec{r}_{ij} - \vec{r}|^3} \rho(r) d^3 r \tag{4.19}$$

where, $m = 0$ in axisymmetric case,

$$\rho(r) = \sum_l f_{jl}(r) P_l(\cos \chi) \tag{4.20}$$

where χ is the angle between $\vec{r}$ and $\langle \vec{u}_j \rangle$, the latter is parallel to the $\vec{e}_z$ axis of symmetry. First we can proof

$$\begin{aligned} \vec{\nabla} \cdot \frac{1}{|\vec{r} - \vec{r}_0|} &= (\vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y} + \vec{e}_z \frac{\partial}{\partial z}) \frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}} \\ &= -\frac{\vec{r} - \vec{r}_0}{|\vec{r} - \vec{r}_0|^3} \end{aligned} \tag{4.21}$$

Using the spin wave function we can get the expectation value of U,

$$\langle \Delta U \rangle = -\sum \sum_{i \neq j} e_i \langle \vec{d}_j \rangle \cdot \int \rho(r) \vec{\nabla} \frac{1}{|\vec{r} - \vec{r}_{ij}|} d^3 r \tag{4.22}$$

We use Legendre's generating function, the references is from "zeng jin yan Quantum Mechanics volume I" page 528

$$\frac{1}{|\vec{r}-\vec{r}_{ij}|} = \left\{ \begin{array}{ll} \frac{1}{r_{ij}}\sum_l (\frac{r}{r_{ij}})^l P_l(\cos\theta), & r < r_{ij} \\ \frac{1}{r}\sum_l (\frac{r_{ij}}{r})^l P_l(\cos\theta), & r > r_{ij} \end{array} \right\}$$ θ is the angle between $\vec{r}$ and $\vec{r}_{ij}$.

If $r < r_{ij}$,

$$\langle \Delta U \rangle = -\sum \sum_{i \neq j} e_i \langle \vec{d}_j \rangle \cdot \int \rho(r) \vec{\nabla} \frac{1}{|\vec{r}-\vec{r}_{ij}|} d^3 r = -\sum \sum_{i \neq j} e_i \langle d_j \rangle S$$

where

$$S = \vec{e}_z \cdot \int \rho(r) \vec{\nabla} \frac{1}{|\vec{r}-\vec{r}_{ij}|} d^3 r$$

Based on the addition theorem of the spherical harmonics

$$P_l(\cos\theta_{12}) = \sum_{m=-l}^{l} Y^*_{lm}(\theta_1, \psi_1) Y^*_{lm}(\theta_2, \psi_2)$$

where $m = 0$ because of the orthogonality.

We know from the above that S can be expanded

$$S = \int \rho(r)\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}_{ij}|} = \int f_{jl}(r)P_{l'}(\cos\chi)[\vec{\nabla}\frac{1}{r_{ij}}\sum_l(\frac{r}{r_{ij}})^l P_l(\cos\theta)]dr^3$$
$$= \int f_{jl}(r)P_{l'}(\cos\chi)[\vec{\nabla}\frac{1}{r_{ij}}\sum_l(\frac{r}{r_{ij}})^l\frac{4\pi}{2l+1}Y_l(\theta_0)Y_l(\chi)]dr^3$$
$$= \frac{1}{r_{ij}^{l+1}}\sum_l\frac{4\pi}{2l+1}Y_l(\theta_0)\int f_{jl}(r)P_{l'}(\cos\chi)[\vec{\nabla}r^lY_l(\chi)]dr^3$$

Making use of the spherical component of $\vec{\nabla}$ in terms of polar basis we can get,where we reference the "Quantum Theory of Angular Momentum" page 8,18 and 147.

$$\nabla_0 r^l Y_{l0}(\chi) = \sqrt{\frac{l^2(2l+1)}{(2l-1)}}\, r^{l-1}Y_{l-1,0}(\chi) \quad (4.23)$$
$$\nabla_{\pm1} r^l Y_{l0}(\chi) = AY_{l-1,\pm1}(\chi) \quad (4.24)$$

where the ∇_0 along z axis. In addition, the orthogonality relation

$$\int Y_{l-1,\pm1}(\chi)Y_{l',0}(\chi)d\Omega = 0 \quad (4.25)$$

so that we get

$$S = \frac{1}{r_{ij}^{l+1}} \sum_{ll'} \frac{4\pi}{2l+1} Y_l(\theta_0) \int f_{jl}(r) P_{l'}(\cos\chi) [\vec{\nabla} r^l Y_l(\chi)] dr^3$$

$$= \vec{e}_z \cdot \sum_{ll'} \frac{4\pi}{2l+1} Y_l(\theta_0) \int P_{l'}(\cos\chi) \sqrt{\frac{l^2(2l+1)}{(2l-1)}} Y_{l-1,0}(\chi) d\Omega \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l+1} dr$$

$$= \vec{e}_z \cdot \sum_{ll'} \frac{4\pi}{2l+1} \sqrt{\frac{l^2(2l+1)}{(2l-1)}} Y_l(\cos\theta_0) \int P_{l'}(\cos\chi) Y_{l-1,0}(\chi) d\Omega \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l+1} dr$$

$$= \vec{e}_z \cdot \sum_{ll'} \frac{4\pi}{2l+1} \sqrt{\frac{l^2(2l+1)}{(2l-1)}} \sqrt{\frac{(2l+1)}{4\pi}} \sqrt{\frac{4\pi}{2l'+1}} P_l(\cos\theta_0) \int Y_{l'0}(\cos\chi) Y_{l-1,0}(\cos\chi)$$

$$d\Omega \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l+1} dr = \sum_{ll'} \frac{4\pi}{2l-1} l P_l(\cos\theta_0) \delta_{l',l-1} \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l+1} dr$$

$$= \vec{e}_z \cdot \sum_{l'} \frac{4\pi}{2l'+1} (l'+1) P_{l'+1}(\cos\theta_0) \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l'+2} dr$$

And at the end we can get,where θ_0 is the angle between $\vec{e}_z$ and $\vec{r}_{ij}$,

$$\langle \Delta U \rangle = -\sum \sum_{i \neq j} e_i \langle d_j \rangle \sum_l \frac{4\pi}{2l+1} (l+1) P_{l+1}(\cos\theta_0) \int_0^{r_{ij}} f_{jl}(r) (\frac{r}{r_{ij}})^{l+2} dr \qquad (4.26)$$

With the same thing if $r > r_{ij}$

$$S' = \int \rho(r)\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}_{ij}|} = \int f_{jl}(r)P_{l'}(\cos\chi)[\vec{\nabla}\frac{1}{r}\sum_l(\frac{r_{ij}}{r})^l P_l(\cos\theta)]dr^3$$

$$= r_{ij}^l \sum_l \frac{4\pi}{2l+1}Y_l(\theta_0)\int f_{jl}(r)P_{l'}(\cos\chi)[\vec{\nabla}r^{-l-1}Y_l(\chi)]dr^3$$

and

$$\nabla_0 r^{-l-1}Y_{l0}(\chi) = -\sqrt{\frac{(l+1)^2(2l+1)}{(2l+3)}}\, r^{-l-2}Y_{l+1,0}(\chi) \quad (4.27)$$

$$\nabla_{\pm1} r^{-l-1}Y_{l0}(\chi) = A' Y_{l-1,\pm1}(\chi) \quad (4.28)$$

So

$$S = -\vec{e}_z\cdot\sum_{ll'}\frac{4\pi}{2l+1}\sqrt{\frac{(l+1)^2(2l+1)}{(2l+3)}}\sqrt{\frac{(2l+1)}{4\pi}}\sqrt{\frac{4\pi}{2l'+1}}$$

$$P_l(\cos\theta_0)\int Y_{l'0}(\cos\chi)Y_{l+1,0}(\cos\chi)d\Omega\int_{r_{ij}}^{\infty} f_{jl}(r)(\frac{r_{ij}}{r})^l dr$$

$$= -\vec{e}_z\sum_{ll'}\frac{4\pi}{2l+3}(l+1)P_l(\cos\theta_0)\delta_{l',l+1}\int_{r_{ij}}^{\infty} f_{jl}(r)(\frac{r}{r_{ij}})^l dr$$

$$= -\vec{e}_z\cdot\sum_{l'}\frac{4\pi}{2l'+1}\, l' P_{l'-1}(\cos\theta_0)\int_{r_{ij}}^{\infty} f_{jl}(r)(\frac{r}{r_{ij}})^{l'-1} dr$$

Last,it is also represented by the same index,

$$\langle U \rangle = -\sum_{i \neq j} e_i \langle d_j \rangle \sum_l \frac{4\pi}{2l+1} \times [(l+1)P_{l+1}(\cos\theta_0) \int_0^{r_{ij}} f_{jl}(r)(\frac{r}{r_{ij}})^{l+2} dr$$
$$-lP_{l-1}(\cos\theta_0) \int_{r_{ij}}^{\infty} f_{jl}(r)(\frac{r}{r_{ij}})^{l-1} dr] \quad (4.29)$$

where $\langle \mu_j \rangle$ is the magnitude of the vector $\langle \boldsymbol{\mu}_j \rangle$. If the zero-field eigenfunction of $T+V_0$ is spherically symmetric, $V|$ will introduce a P_1-type dependence on the angle between $\mathbf{r}_{ij}$ and $\mathbf{E}$, so that only the $P_1(\cos\theta)$ terms in Eq. (12) will contribute. Thus only f_{j0} and f_{j2} appear in the spin-field interaction.[16] Numerical results based on Eq. (12) will be presented in Sec. VII.

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The expectation value of $\langle \Delta U \rangle$ given by Eq. (12) easily calculated for a hydrogen atom in its grour state. In Eq. (12), we put $\langle \mu_j \rangle = \langle \mu \rangle$ for the nucleu $e_i = -e$ for the electron, and $\mathbf{r}_{ij} = \mathbf{r}_1$ for the vector fro nucleus to electron. The normalized wave functio correct to first order in V, has been given by Kotani[1]

$$u_0 = (\pi a_0^3)^{-\frac{1}{2}} e^{-r_1/a_0} [1 - (\mathbf{r}_1 \cdot \mathbf{E}/2e)(r_1 + 2a_0)],$$
$$a_0 = \hbar^2/me^2. \quad (1$$

Eq. (12) that contributes to the spin-field interaction is

$$4\pi e \langle \mu \rangle \cos\theta \left[\int_0^{r_1} (r/r_1)^2 f_0(r) dr - \int_r^{\infty} (2r_1/5r) f_2(r) dr \right]. \quad (16)$$

The expectation value of (16) with the wave function (15) is readily calculated to be

$$(16\pi/3a_0^2)(\langle \boldsymbol{\mu} \rangle \cdot \mathbf{E}) \int_0^{\infty} r^4 [f_0(r) + (4/25) f_2(r)] dr. \quad (17)$$

We have made use of the fact that $\int_0^\infty r^2 f_0(r) dr = 0$ since the volume integral of (11) is zero, and have also assumed that the spatial extent of f_0 and f_2 is much smaller than a_0. The expression (17) may be written in terms of moments of the difference distribution function $\rho(\mathbf{r})$ given by Eqs. (9) and (11):

$$R_l^2 = \int r^2 P_l(\cos\chi) \rho(\mathbf{r}) d^3r = (4\pi/2l+1) \int_0^\infty r^4 f_l(r) dr.$$

Thus the spin-field interaction energy is

$$[4(\langle \boldsymbol{\mu} \rangle \cdot \mathbf{E})/3a_0^2][R_0^2 + \tfrac{4}{5} R_2^2]. \quad (18)$$

Recent experiments of Collard *et al.*[18] in which high energy electrons are scattered from He^3, give 1.97×10^{-13} cm for the root-mean-square (rms) radius of the charge density of the He^3 nucleus, and 1.69×10^{-13} cm for the rms radius of the magnetic moment density. If it is assumed that whatever electric dipole moment is present is distributed in the same way as the magnetic dipole moment, then

$$R_0^2=[(1.97)^2-(1.69)^2]\times10^{-26}\ \text{cm}^2=1.02\times10^{-26}\ \text{cm}^2.$$

Nothing is known of R_2, and it may safely be presumed to be negligibly small. We thus expect the spin-field interaction energy in He^3 that arises from the finite size of the nucleus to be roughly equal to $1.4\times10^{-9}\times(\langle\boldsymbol{\mu}\rangle\cdot\mathbf{E})$.

For helium, we use the approximate wave function (19) with $Z=27/16$. In (14), e_0 must now be replaced by $2e$, and $\mathbf{p}=-\mathbf{p}_1-\mathbf{p}_2$; there are, however, no cross terms between the $i=1$ and $i=2$ parts of the expectation value of (14). The interaction then turns out to be $\frac{1}{2}Z^2$ times the hydrogen value (20). With $\kappa=-2.127$ for He^3, the spin-field interaction energy is roughly equal to $-1.5\times10^{-7}(\langle\boldsymbol{\mu}\rangle\cdot\mathbf{E})$, or about a hundred times larger than the finite size effect.

From a classical point of view, there can be no average electric field at the nucleus unless some non-electric force is available to keep the nucleus from accelerating under the influence of this electric field. In the finite size effect, this force is supplied by the non-electric interactions between nucleons and mesons. These give the nucleus a finite size, and make it possible for whatever electric dipole moment it may possess to be in a region where the electric field is not exactly zero.

If the parameter γ of Sec. I is 10^{-7} and E is 10^5 V/cm, the precession rate of He^3 nuclei caused by the magnetic moment effect is roughly half a degree per day. It seems possible that considerably smaller precession rates can be measured.[8]

Schiff Moment Derivation (part I)

The atomic Hamiltonian in the uniform external field $\vec{E}_{\text{ext}}$

$$\hat{H}_{\text{Atom}} = \hat{H}_{\text{Electrons}} + \hat{H}_{\text{Nucleus}} + \sum_{i=1}^{Z} \left(-e\Phi(\vec{r}_i) + e\vec{r}_i\vec{E}_{\text{ext}} \right) - \vec{d}_N\vec{E}_{\text{ext}},$$

where $\Phi(\vec{r})$ is the nuclear electrostatic potential, d_N is nuclear EDM

Let's use the following unitary transformation

$$\hat{H}'_{\text{Atom}} = \mathrm{e}^{i\hat{U}}\hat{H}_{\text{Atom}}\mathrm{e}^{-i\hat{U}} \approx \hat{H}_{\text{Atom}} + i[\hat{U}, \hat{H}_{\text{Atom}}]$$

with $\hat{U}$ taken as (**note that $\langle\vec{d}_N\rangle$ below is not a operator**)

$$\hat{U} = \frac{\langle\vec{d}_N\rangle}{Ze}\sum_{i=1}^{Z}\hat{\vec{p}}_i$$

we exclude the nuclear electric dipole moment from H_{Atom} completely.

Schiff Moment Derivation (part II)

The net E-field on the nucleus vanishes on average:

$$i[\hat{U}, \hat{H}_{\text{Atom}}] = \langle \vec{d}_N \rangle \left(\vec{E}_{\text{ext}} - \frac{1}{Z} \sum_{i=1}^{Z} \vec{\nabla}_i \Phi(\vec{r}_i) \right) = \langle \vec{d}_N \rangle \left(\vec{E}_{\text{ext}} + \vec{E}_{\text{due electrons}} \right)$$

The new "rotated" Hamiltonian $\hat{H}'_{\text{Atom}} \approx \hat{H}_{\text{Atom}} + i[\hat{U}, \hat{H}_{\text{Atom}}]$ will have two terms: the first vanishes

$$\langle N| \left(\langle \vec{d}_N \rangle - \vec{d}_N \right) \vec{E}_{\text{ext}} |N\rangle = 0,$$

the second one gives the following effective potential

$$\langle N| - e\Phi(\vec{r}) - \frac{1}{Z} \langle \vec{d}_N \rangle \vec{\nabla} \Phi(\vec{r}) |N\rangle = -\frac{Ze^2}{|\vec{r}\,|} - 4\pi e \vec{S} \vec{\nabla} \delta(\vec{r}) + \dots$$

Schiff Moment Derivation (part III)

Hyper-virial theorem: on average, a neutral atom, placed in the uniform external electric field, does not move. It means that, on average, total electric field acting on every charge inside the atomic system should vanish

Schiff moment: the first non-vanishing dipole term

$$\hat{S}_k = \frac{1}{10}\int \left(x^2 x_k - \frac{5}{3}\langle x^2\rangle_{ch} x_k - \frac{2}{3}\langle Q_{kk'}\rangle x_{k'} \right) \rho(\vec{x}) d^3x$$

The Schiff moment produces the $\mathcal{P}, \mathcal{T}$-odd electrostatic potential

$$h(\vec{r}) = 4\pi \vec{S} \cdot \vec{\nabla}\delta(\vec{r})$$

that induces the atomic EDM

$$\delta\varphi(\mathbf{R}) = e\int \frac{\delta\rho(\mathbf{r})\, d^3r}{|\mathbf{R}-\mathbf{r}|} + \frac{1}{Z}(\mathbf{d}\nabla)\int \frac{\rho_0(r)\, d^3r}{|\mathbf{R}-\mathbf{r}|},$$


Thank You