Resonant and scattering states in the nonlocalized cluster model

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DB and Zhongzhou Ren, PRC **101**, 034311 (2020).

Hantao Zhang, **DB**, Zhen Wang, and Zhongzhou Ren, PRC **105**, 054317 (2022).

Virtual Seminar at School of Physics and Astronomy, Sun Yat-Sen University

Outline

1. Nuclear clustering
   - What's cluster states? Why it is important?
   - What's nonlocalized clustering?
   - Why resonant and scattering states?
2. Theoretical formalism
   - Nonlocalized cluster model
   - $\cdots +$ the R-matrix theory
   - $\cdots +$ complex scaling method (CSM)
3. The $\alpha + \alpha$ system as a proof of concept
   - The bound-state approximation
   - The R-matrix results
   - The CSM results
4. Summary

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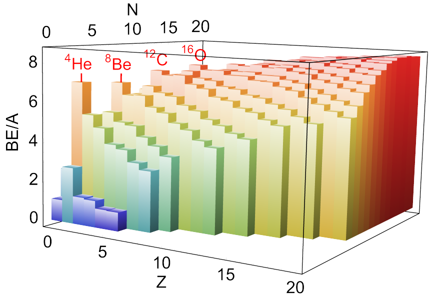
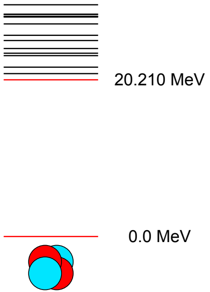
Nuclear clustering: Phenomenological viewpoint I

Nuclei are **self-bound** systems made of protons and neutrons.



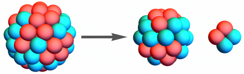


***α* clusters** could be taken as effective building blocks, e.g.,
$^8\text{Be} \approx \alpha + \alpha$, $^{12}\text{C} \approx \alpha + \alpha + \alpha$, $^{16}\text{O} \approx \alpha + \alpha + \alpha + \alpha, \cdots.$

This picture could also be extended to the **heavier** clusters, e.g.,
$^{16}\text{O} \approx \alpha + {}^{12}\text{C}$, $^{20}\text{Ne} \approx \alpha + {}^{16}\text{O}$, $^{44}\text{Ti} \approx \alpha + {}^{40}\text{Ca}, \cdots.$

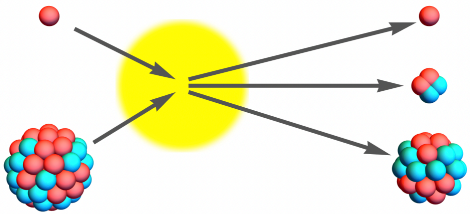
Nuclear clustering: Phenomenological viewpoint II

○ **Cluster radioactivity**: spontaneous emissions of α particles, ^{14}C, ^{20}O, ^{24}Ne, $\cdots$ from heavy nuclei.



A natural explanation: the emitted cluster first **preforms** inside the **unstable** parent nucleus and then **escapes** via quantum tunneling
$\Rightarrow$ Nuclear clustering in **radioactive** heavy nucleus (任老师团队)

○ **What about the stable heavy nucleus?** Cluster-knockout reaction.

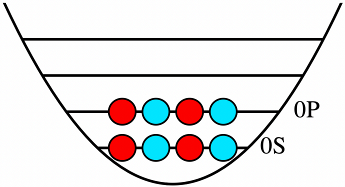


Nuclear clustering: Theoretical viewpoint

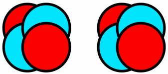
Consider a nuclear many-body Schrödinger equation

$$\left(T + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} \right) |\Psi\rangle = E\,|\Psi\rangle\,,$$

and we want to make **approximations** to the exact wave function $|\Psi\rangle$.



Nuclear Shell Model


Cluster Model

○ **Which model is better?**

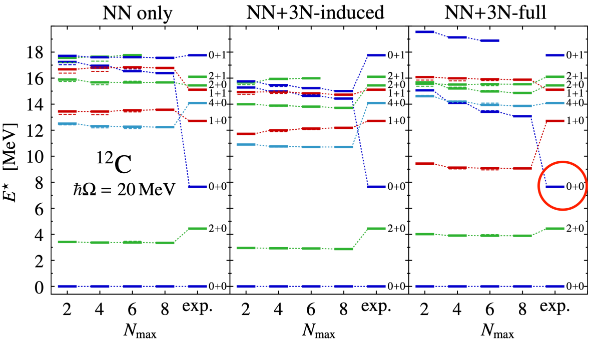
○ Compare $\langle \mathcal{O} \rangle_{\text{Model}} \equiv \langle \text{Model}|\mathcal{O}|\text{Model}\rangle$ and $\langle \mathcal{O} \rangle_{\Psi} \equiv \langle \Psi|\mathcal{O}|\Psi\rangle$; the closer, the better. Often, systematical improvements can be made upon the good models.

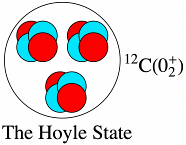
○ **Cluster state**: the cluster configurations are crucial, especially when $(\langle \mathcal{O} \rangle_{\text{CM}} - \langle \mathcal{O} \rangle_{\Psi})^2 \ll (\langle \mathcal{O} \rangle_{\text{SM}} - \langle \mathcal{O} \rangle_{\Psi})^2$.

It would be nice if there is one model killing all the problems.

Shell model was thought to be a candidate. (Un)Fortunately, life is not that easy!

"Rebellious sons": the Hoyle state, cluster decays from heavy elements



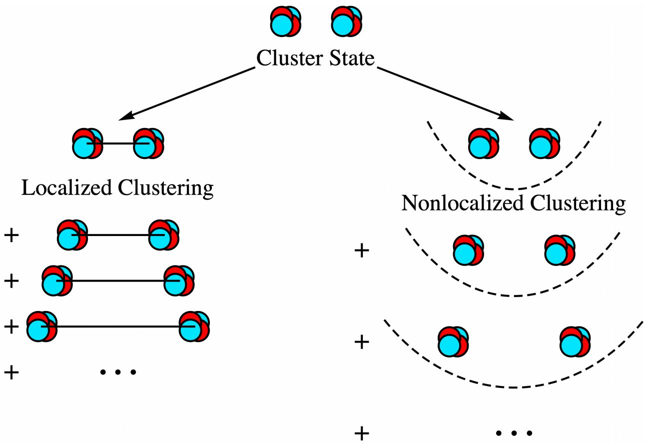


R. Roth *et al.*, PRL **107**, 072501 (2011): NCSM + SRG + chiral potentials

Localized Clustering *vs* Nonlocalized Clustering

How to model a cluster state?

Building blocks: clusters made of nucleons

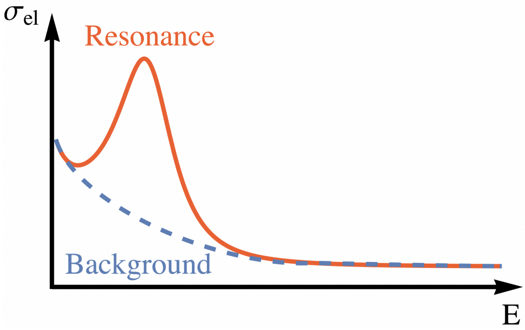


Nonlocalized cluster model is proposed by 周波, Y. Funaki, H. Horiuchi, 任老师 *etc*, PRL **110**, 262501 (2013), often a better starting point.

Consider the elastic scattering $a + A \to a + A$. Its cross section is given by

$$\sigma_{el} = \frac{4\pi}{k^2} \sum_{L=0}^{\infty} (2L+1) \sin^2 \delta_L .$$

Naïvely, $k \uparrow$, $E \uparrow$, $\sigma_{el} \downarrow$. The presence of resonance invalidates this expectation.



Resonant and Scattering States: Theoretical Viewpoint

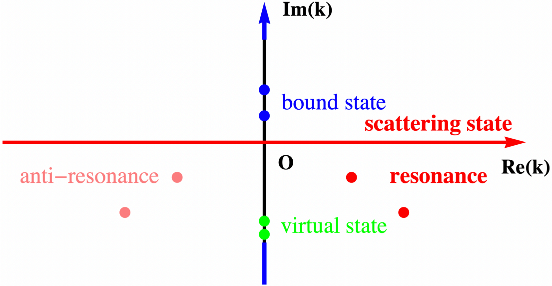
Boundary conditions of Schrödinger equations

$$-\frac{d^2}{dr^2}\phi(r) + V(r)\phi(r) = k^2\phi(r).$$

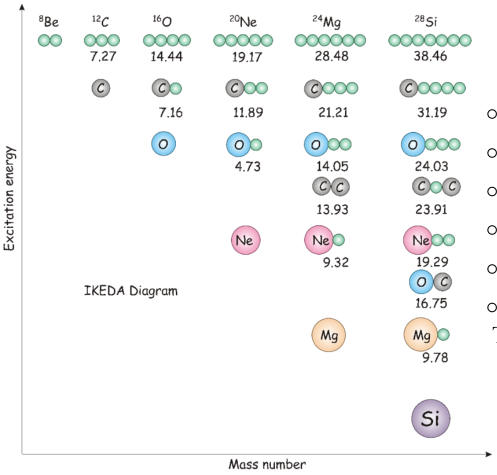
$V(r)$ is short-range $\Rightarrow \phi(r) \to A\exp(-ikr) + B\exp(ikr)$, as $r \to \infty$.

- **Scattering state**: $k^2 > 0$, $\phi_S(r) \to \exp(-ikr) + S(k)\exp(ikr)$
- Bound state: $k^2 < 0$, $\phi_B(r) \to B\exp(-\kappa r)$, $\kappa = |k|$.
- **Resonant state**: $\mathrm{Re}\,k > 0$, $\mathrm{Im}\,k < 0$, $\phi_R(r) \to \exp(ikr) \propto \exp(i\mathrm{Re}\,k\,r)$ $\times\exp(-\mathrm{Im}\,k\,r)$, blowing up in infinity,
  $\boldsymbol{\mathcal{E}_{\rm res} = E - i\Gamma/2 = |\widetilde{E}|\exp(-2i\theta_R) \Rightarrow \theta_R = \frac{1}{2}\arctan\left(\frac{\Gamma}{2E}\right)}$.

Pole structures of *S*-matrix



Why the resonant state is important for nuclear clustering?



- $^8\mathrm{Be}(0_1^+)$ just above 2α threshold
- $^{12}\mathrm{C}(0_2^+)$ just above 3α threshold
- $^{16}\mathrm{O}(1_2^-)$ above $\alpha + {}^{12}\mathrm{C}$ threshold
- $^{16}\mathrm{O}(0_6^+)$ just above 4α threshold
- $^{20}\mathrm{Ne}(1_1^-)$ above $\alpha + {}^{16}\mathrm{O}$ threshold
- $\cdots$

These cluster states are **resonances**.

Bound-state approximation: suitable for resonant cluster states with narrow widths

Pros:

1. intuitive physical picture
2. bound-state codes reusable

Cons:

1. not easy to **distinguish between resonant and scattering states**, especially for broad resonances
2. physical picture sometimes inaccurate

○ $^{8}\mathrm{Be}(2_1^+)$, a member of the ground-state band of $^{8}\mathrm{Be}$, has a large decay width ~ 3 MeV compared to its total energy ~ 1.5 MeV above the 2α threshold.

○ $^{12}\mathrm{C}(0_3^+)$, which is conjectured to be **a breathing mode of the Hoyle state** $^{12}\mathrm{C}(0_2^+)$, has a large decay width ~ 1.45 MeV compared to its total energy ~ 1.77 MeV above the 3α threshold.

$\Rightarrow$ A proper treatment of their resonant nature is important.

Goal I

Improving nonlocalized cluster model for resonances.

Why study the scattering states in microscopic cluster models?

Main reasons:

1. Scattering states are important ingredients in nuclear reaction theories, e.g., elastic scattering, breakup reaction (CDCC: continuum-discretized coupled channels), etc.
2. Microscopic reaction models are less explored than structural models. The initial and final states naturally contain "clusters" ⇒ Microscopic cluster models play a role naturally.
3. Interesting by itself, potential applications in nuclear astrophysics.

Goal II

Studying the nucleus-nucleus elastic scattering in nonlocalized cluster model.

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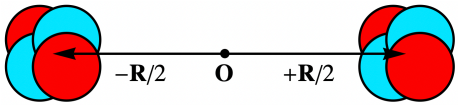
The α-cluster wave function

$$\Phi_\alpha(\boldsymbol{R}) = \frac{1}{\sqrt{4!}}\det\{\varphi_{0s}(\boldsymbol{r}_1 - \boldsymbol{R})\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}(\boldsymbol{r}_4 - \boldsymbol{R})\chi_{\sigma_4\tau_4}\},$$

$$\varphi_{0s}(\boldsymbol{r}) = (\pi b^2)^{-3/4}\exp\left[-\frac{\boldsymbol{r}^2}{2b^2}\right].$$

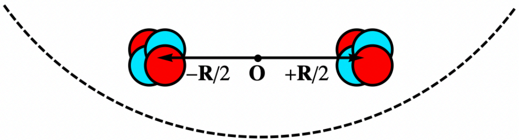
Brink wave function

$$\Phi_{\rm B}(\boldsymbol{R}) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{8!}}\det\{\varphi_{0s}(\boldsymbol{r}_1 - \boldsymbol{R}/2)\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}(\boldsymbol{r}_4 - \boldsymbol{R}/2)\chi_{\sigma_4\tau_4}$$
$$\times\,\varphi_{0s}(\boldsymbol{r}_5 + \boldsymbol{R}/2)\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}(\boldsymbol{r}_8 + \boldsymbol{R}/2)\chi_{\sigma_4\tau_4}\}.$$



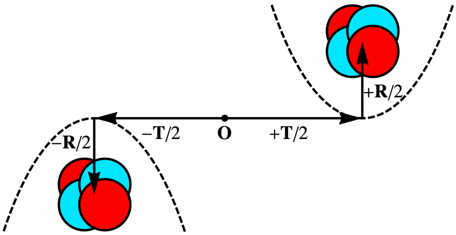
Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function

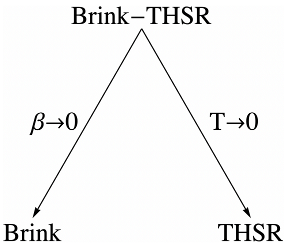
$$\Psi(\beta) = \mathcal{N} \int \mathrm{d}^3 R \exp\left(-\frac{\boldsymbol{R}^2}{2\beta^2}\right) \Phi_{\mathrm{B}}(\boldsymbol{R}).$$



Brink-THSR wave function

$$\Psi(\beta, \boldsymbol{T}) = \mathcal{N} \int \mathrm{d}^3 R \exp\left(-\frac{\boldsymbol{R}^2}{2\beta^2}\right) \Phi_{\mathrm{B}}(\boldsymbol{R} + \boldsymbol{T})$$





Separation of center-of-mass motion

$$\Psi(\beta, \mathbf{T}) = \Psi_{\mathrm{CM}}(\mathbf{X}_{\mathrm{CM}}) \times \widehat{\Psi}(\beta, \mathbf{T}),$$

$$\Psi_{\mathrm{CM}}(\mathbf{X}_{\mathrm{CM}}) = \left(\frac{8}{\pi b^2}\right)^{3/4} \exp\left(-\frac{4\mathbf{X}_{\mathrm{CM}}^2}{b^2}\right),$$

$$\widehat{\Psi}(\beta, \mathbf{T}) = \frac{1}{\sqrt{140}} \mathscr{A}_{12} \left[\Gamma(\boldsymbol{\rho}, \beta, \mathbf{T}) \widehat{\phi}(\alpha_1) \widehat{\phi}(\alpha_2)\right] \Rightarrow u(\boldsymbol{\rho}),$$

$$\Gamma(\boldsymbol{\rho}, \beta, \mathbf{T}) = \left(\frac{2}{\pi}\right)^{3/4} \frac{b^{3/2}}{(b^2 + 2\beta^2)^{3/2}} \exp\left[-\frac{(\boldsymbol{\rho} - \mathbf{T})^2}{b^2 + 2\beta^2}\right].$$

Angular momentum projection

$$\Psi(\beta, \mathbf{T}) = \Psi_{\mathrm{CM}}(\mathbf{X}_{\mathrm{CM}}) \times 4\pi \sum_{LM} \widehat{\Psi}_{LM}(\beta, T) Y_{LM}^*(\Omega_T),$$

$$\widehat{\Psi}_{LM}(\beta, T) = \frac{1}{\sqrt{140}} \mathscr{A}_{12} \Gamma_L(\rho, \beta, T) Y_{LM}(\Omega_\rho) \widehat{\phi}(\alpha_1) \widehat{\phi}(\alpha_2) \Rightarrow \frac{u_L(\rho)}{\rho} Y_{LM}(\Omega_\rho),$$

$$\Gamma_L(\rho, \beta, T) = \left(\frac{2}{\pi}\right)^{3/4} \frac{b^{3/2}}{(b^2 + 2\beta^2)^{3/2}} \exp\left(-\frac{\rho^2 + T^2}{b^2 + 2\beta^2}\right) \mathrm{i}_L\left(\frac{2\rho T}{b^2 + 2\beta^2}\right).$$

The Hamiltonian is given by

$$H = T - T_{\mathrm{CM}} + V_N + V_C,$$

$$T - T_{\mathrm{CM}} = -\sum_{i=1}^{A} \frac{1}{2m} \left(\frac{\partial}{\partial \boldsymbol{r}_i} \right)^2 + \frac{1}{2Am} \left(\frac{\partial}{\partial \boldsymbol{X}_{\mathrm{CM}}} \right)^2 .$$

Effective nucleon-nucleon interactions are adopted in nonlocalized cluster models, as well as many other microscopic cluster models.

$$V_{N,ij}(r) = \sum_{k=1}^{N_g} V_k \exp\left[-(r/a_k)^2\right] \left(w_k - m_k P_{ij}^{\sigma} P_{ij}^{\tau} + b_k P_{ij}^{\sigma} - h_k P_{ij}^{\tau} \right) .$$

Interaction	k	V_k (MeV)	a_k (fm)	w_k	m_k	b_k	h_k
Volkov No. 1	1	−83.34	1.60	$1-M$	M	0	0
	2	144.86	0.82	$1-M$	M	0	0
Minnesota	1	200	$1/\sqrt{1.487}$	$u/2$	$1-u/2$	0	0
	2	-178	$1/\sqrt{0.639}$	$u/4$	$1/2-u/4$	$u/4$	$1/2-u/4$
	3	-91.85	$1/\sqrt{0.465}$	$u/4$	$1/2-u/4$	$-u/4$	$u/4-1/2$

The Bound-State Approximation

Single basis wave function:

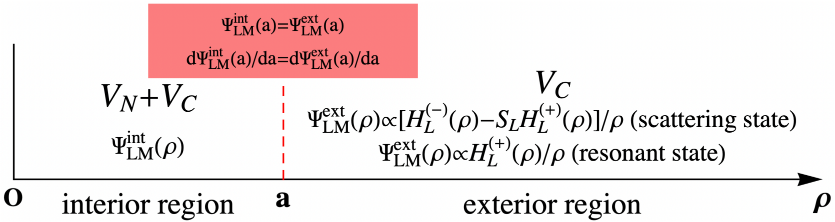
- THSR: $E_L(\beta) = \min \dfrac{\langle \widehat{\Psi}_{LM}(\beta) | H_L | \widehat{\Psi}_L M(\beta) \rangle}{\langle \widehat{\Psi}_{LM}(\beta) | \widehat{\Psi}_{LM}(\beta) \rangle}.$
- Brink-THSR: $E_L(\beta, T) = \min \dfrac{\langle \widehat{\Psi}_{LM}(\beta, T) | H_L | \widehat{\Psi}_{LM}(\beta, T) \rangle}{\langle \widehat{\Psi}_{LM}(\beta, T) | \widehat{\Psi}_{LM}(\beta, T) \rangle}.$

Multiple basis wave functions:

- THSR: $\widetilde{\Psi}_{LM} = \displaystyle\int \mathrm{d}\beta f(\beta)\, \widehat{\Psi}_{LM}(\beta).$
- Brink-THSR: $\widetilde{\Psi}_{LM}(\beta) = \displaystyle\int \mathrm{d}T f(T)\, \widehat{\Psi}_{LM}(\beta, T).$

$\Rightarrow$ variational principle $\Rightarrow$ Hill-Wheeler equation.

$\cdots$ + the R-Matrix Theory



$$\eta = Z_\alpha^2 e^2 \sqrt{\frac{\mu}{2E}} \text{ Coulomb-Sommerfeld parameter}$$

$$\widehat{\Psi}^{\text{int}}_{LM}(E) = \int \mathrm{d}T f_L(T,E)\widehat{\Psi}_{LM}(\beta,T) = \sum_n \widetilde{f}_L(T_n,E)\widehat{\Psi}_{LM}(\beta,T_n),$$

$$\widehat{\Psi}^{\text{ext}}_{LM}(E) = \frac{1}{\sqrt{35}} g_L^{\text{ext}}(\rho) Y_{LM}(\Omega_\rho)\widehat{\phi}(\alpha_1)\widehat{\phi}(\alpha_2),$$

$$\Rightarrow (H_L + \mathcal{L}(B) - E)\Psi^{\text{int}}_{LM} = \mathcal{L}(B)\Psi^{\text{ext}}_{LM} \quad \text{Bloch-Schrödinger equation}$$

$$\mathcal{L}(B) = 35\frac{1}{2\mu a}\delta(\rho - a)\left(\frac{\mathrm{d}}{\mathrm{d}\rho}\rho - B\right)$$

Scattering state: $B = 0$

$$\sum_{n'} [C(0,E)]_{nn'} \widetilde{f}_L(T_{n'},E) = \langle \widehat{\Psi}_L(\beta,T_n)|\mathcal{L}(0)|\widehat{\Psi}_L^{\rm ext}(E)\rangle \,,$$

$$[C(0,E)]_{nn'} = \left(\widehat{\Psi}_L(\beta,T_n)\middle| H_L + \mathcal{L}(0) - E \middle| \widehat{\Psi}_L(\beta,T_{n'})\right) ,$$

$$\mathcal{R}_L = \frac{a}{2\mu} \sum_{nn'} \Gamma_L(a,\beta,T_n)[C(0,E)]^{-1}_{nn'} \Gamma_L(a,\beta,T_{n'}),$$

$$\mathcal{S}_L = \frac{\mathcal{H}_L^{(-)}(\eta,ka) - ka\,\mathcal{H}_L^{(-)'}(\eta,ka)\mathcal{R}_L}{\mathcal{H}_L^{(+)}(\eta,ka) - ka\,\mathcal{H}_L^{(+)'}(\eta,ka)\mathcal{R}_L}.$$

Resonant state: $B = B_* \equiv ka \frac{\mathcal{H}_L^{(+)'}(\eta,ka)}{\mathcal{H}_L^{(+)}(\eta,ka)}$

$$\sum_{n'} \left(\widehat{\Psi}_L(\beta,T_n)|H_L + \mathcal{L}(B_*)|\widehat{\Psi}_L(\beta,T_{n'})\right) \widetilde{f}_L(T_{n'},E)$$

$$= E \sum_{n'} \left(\widehat{\Psi}_L(\beta,T_n)|\widehat{\Psi}_L(\beta,T_{n'})\right) \widetilde{f}_L(T_{n'},E).$$

self-consistent generalized eigenvalue problem

$\cdots$ + Complex Scaling Method (CSM)

Complex scaling transformation

$$\boldsymbol{r} \to \boldsymbol{r}\exp(i\theta), \qquad \psi(\boldsymbol{r}) \to \exp\left(i3\theta/2\right)\phi(\boldsymbol{r}\exp(i\theta)).$$

$$\Rightarrow \text{similarity transformation } S(\theta) = \prod_j \left[\exp(3i\theta/2)\exp(i\theta\boldsymbol{r}_j \cdot \nabla_j)\right],$$

$$H \to S(\theta)HS(\theta)^{-1}, \qquad \Psi \to S(\theta)\Psi.$$

The Schrödinger equation:

$$-\frac{\mathrm{d}^2}{\mathrm{d}r^2}\phi(r) + V(r)\phi(r) = k^2\phi(r)$$

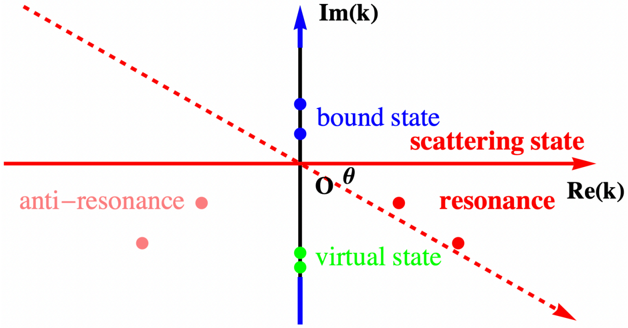
$$\Rightarrow -\exp(-2i\theta)\frac{\mathrm{d}^2}{\mathrm{d}r^2}\phi_\theta(r) + V(r\exp(i\theta))\phi_\theta(r) = k^2\phi_\theta(r).$$

Square integrable eigensolutions:

∘ Bound state: $\phi_{\mathrm{B}}(r) \propto \exp(-\kappa r) \to \phi_{\mathrm{B}}^{\theta}(r) \propto \exp(-\kappa r\exp(i\theta))$

∘ Resonant state: $\phi_{\mathrm{R}}(r) \propto \exp(i|k|\exp(-i\theta_{\mathrm{R}})r) \Rightarrow$
$\phi_{\mathrm{R}}^{\theta}(r) \propto \exp(i|k|r\exp(-i\theta_{\mathrm{R}} + i\theta))$; when $\theta > \theta_R$, $\phi_{\mathrm{R}}^{\theta}(r) \to 0$ as $r \to \infty$.

∘ Scattering state: $\phi_{\mathrm{R}}(r) \propto A\exp(-ikr) + B\exp(ikr) \Rightarrow$
$\phi_{\mathrm{R}}^{\theta}(r) \propto A\exp(-ikr\exp(i\theta)) + B\exp(ikr\exp(i\theta))$; when $k \to k\exp(-i\theta)$,
square integrable, discretized energy.



$\Leftarrow$ **ABC theorem** by **A**guilar, **C**ombes, and **B**alslev.

$\circ$ $H \to H_\theta = \exp(-2i\theta)(T - T_{\rm CM}) + \sum_{i<j} \left[V_N(r_{ij}\exp(i\theta)) + V_C(r_{ij}\exp(i\theta))\right],$

$\circ$ **The complex scaled α-cluster wave function**

$$\Phi_\alpha^\theta(\boldsymbol{R}) = \frac{1}{\sqrt{4!}}\det\{\varphi_{0s}^\theta(\boldsymbol{r}_1 - \boldsymbol{R})\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}^\theta(\boldsymbol{r}_4 - \boldsymbol{R})\chi_{\sigma_4\tau_4}\},$$

$$\varphi_{0s}^\theta(\boldsymbol{r}) = (\pi b^2\exp(-2i\theta))^{-3/4}\exp\left[-\frac{\boldsymbol{r}^2}{2b^2\exp(-2i\theta)}\right].$$

○ Complex scaled Brink wave function

$$\Phi_{\mathrm{B}}^{\theta}(\boldsymbol{R}) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{8!}}\det\{\varphi_{0s}^{\theta}(\boldsymbol{r}_1 - \boldsymbol{R}/2)\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}^{\theta}(\boldsymbol{r}_4 - \boldsymbol{R}/2)\chi_{\sigma_4\tau_4}$$
$$\times\, \varphi_{0s}^{\theta}(\boldsymbol{r}_5 + \boldsymbol{R}/2)\chi_{\sigma_1\tau_1}\cdots\varphi_{0s}^{\theta}(\boldsymbol{r}_8 + \boldsymbol{R}/2)\chi_{\sigma_4\tau_4}\}.$$

Doing the substitution $\boldsymbol{r} \to \boldsymbol{r}\exp(-i\theta)$ in the matrix elements

$$H_\theta \to H,$$
$$\varphi_{0s}^{\theta}(\boldsymbol{r}_1 - \boldsymbol{R}/2) \to \exp(-i3\theta/2)\varphi_{0s}^{\theta}(\boldsymbol{r}\exp(-i\theta) \mp \boldsymbol{R}/2),$$
$$= (\pi b^2)^{-3/4}\exp\left[-\frac{(\boldsymbol{r} - \boldsymbol{R}\exp(i\theta))^2}{2b^2}\right].$$

Therefore, the complex scaling transformation can be done by replacing $\Phi_{\mathrm{B}}(\boldsymbol{R}) \to \Phi_{\mathrm{B}}^{\theta}(\boldsymbol{R}) \equiv \Phi_{\mathrm{B}}^{\theta}(\boldsymbol{R}\exp(i\theta))$, while keeping the Hamiltonian not transformed.

○ Complex scaled THSR wave function

$$\Psi(\beta) \to \Psi_\theta(\beta) \equiv \mathcal{N}\int \mathrm{d}^3R\exp\left(-\frac{\boldsymbol{R}^2}{2\beta^2}\right)\Phi_{\mathrm{B}}(\boldsymbol{R}\exp(i\theta))$$
$$= \mathcal{N}'\int \mathrm{d}^3R\exp\left(-\frac{\boldsymbol{R}^2}{2[\beta\exp(i\theta)]^2}\right)\Phi_{\mathrm{B}}(\boldsymbol{R}).$$

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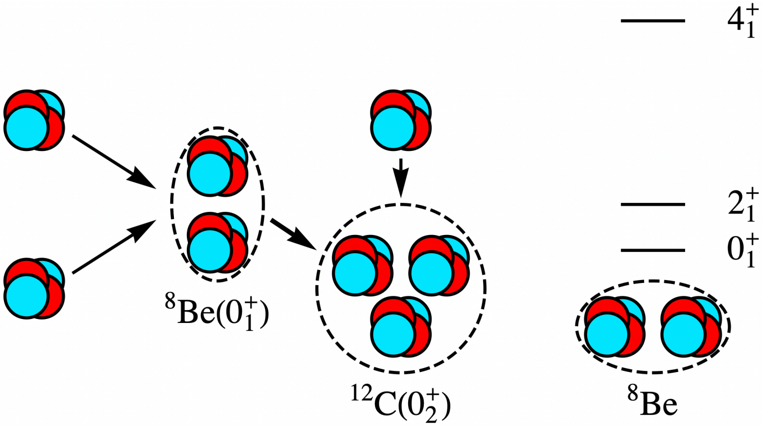
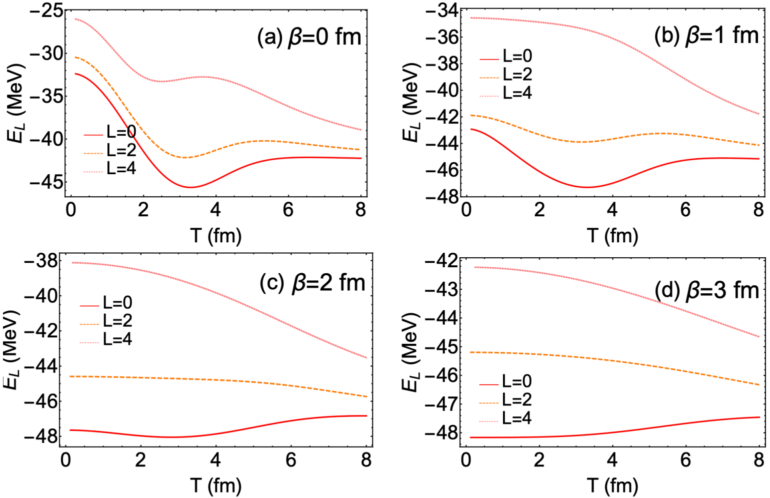
Triple-α process in stars

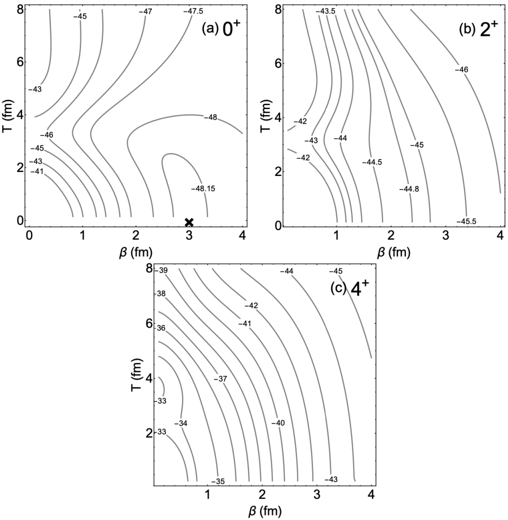


The $\alpha + \alpha$ scattering is the background process.

The Bound-State Approximation



The energy curves for the 0^+, 2^+, and 4^+ states of ^{8}Be given by a single Brink-THSR wave function, with the parameter β being 0 fm, 1 fm, 2 fm, and 3 fm. For $\beta = 0$ fm, the Brink-THSR wave function is reduced to the Brink wave function.



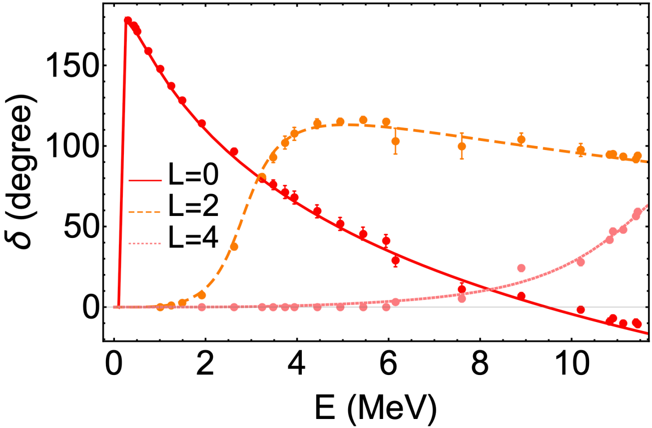
The energy surfaces for the 0^+, 2^+, and 4^+ states of ^{8}Be given by a single Brink-THSR wave function.

^{8}Be

Iteration solutions of the Bloch-Schrödinger equation for the low-lying resonant states of ^{8}Be.

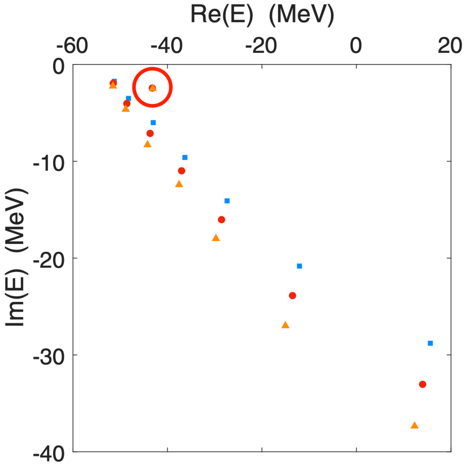
Iterations	$L=0$	$L=2$	$L=4$
1	0.1	3	$20-2.5i$
2	$0.09622-8.1197\times10^{-6}i$	$2.9692-0.5601i$	$11.6042-1.1359i$
3	$0.09700-4.1253\times10^{-6}i$	$2.8748-0.6641i$	$11.3162-2.2401i$
...	...	...	...
12	$0.09687-5.0984\times10^{-6}i$	$2.8190-0.6636i$	$11.8490-2.2588i$
13	$0.09687-5.0984\times10^{-6}i$	$2.8190-0.6636i$	$11.8460-2.2579i$
14	...	...	$11.8459-2.2599i$
...	...	...	...
20	...	...	$11.8466-2.2594i$
21	...	...	$11.8466-2.2594i$

The $\alpha + \alpha$ scattering



The phase shifts for the $\alpha + \alpha$ elastic scattering in the S, D, and G waves against the total energy of the $\alpha + \alpha$ system in the CM frame. The parameter β takes the values of 0 fm, 0.5 fm, and 1 fm. The channel radius is given by $a = 7.0$ fm.

The CSM Results



Complex energy spectrum for the 4^+ state with THSR wave functions. The blue square, red circle, and orange triangle correspond to $\theta = 20^\circ, 23^\circ, 26^\circ$ respectively.

Outline

1. Nuclear clustering
   - What's cluster states? Why it is important?
   - What's nonlocalized clustering?
   - Why resonant and scattering states?
2. Theoretical formalism
   - Nonlocalized cluster model
   - $\cdots +$ the R-matrix theory
   - $\cdots +$ complex scaling method (CSM)
3. The $\alpha + \alpha$ system as a proof of concept
   - The bound-state approximation
   - The R-matrix results
   - The CSM results
4. Summary

Summary

○ Nonlocalized cluster model is a microscopic model for nuclear cluster physics based on the picture of nonlocalized clustering.

○ Hybridize **nonlocalized cluster model** with **the *R*-matrix theory** and **CSM** to study resonant and scattering states.

○ The spectrum properties of ^{8}Be and the $\alpha + \alpha$ elastic scattering are taken to validate the hybrid models, well consistent with other methods.


Thank you!
河海大学